

MOLNÉVS
ARITHMETIC
PART II.

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MOLINEUX'S
ARITHMETIC.

PART THE SECOND.

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ARITHMETIC.

PART THE SECOND.

LONDON:
SPOTTISWOODE and SHAW,
New-street-Square.

AN
INTRODUCTION
TO
PRACTICAL ARITHMETIC:

IN TWO PARTS.

WITH VARIOUS NOTES,

AND

OCCASIONAL DIRECTIONS,

FOR THE USE OF LEARNERS.

PART THE SECOND.

BY THOMAS MOLINEUX,

LATE TEACHER OF ACCOUNTS, SHORT-HAND, AND MATHEMATICS,
AT THE FREE GRAMMAR-SCHOOL, IN MACCLESFIELD.

ARITHMETIC is one of the most useful inventions that the ingenuity of man has discovered; and the study of it ought to be cultivated with the greatest assiduity, as it improves the memory, introduces a habit of precision and correctness, and is the basis of all commercial transactions.

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ADVERTISEMENT.

THE former Part of this Work, containing all the fundamental Rules of Practical Arithmetic, having been favourably received by the Public, the Author, at the request of several respectable Schoolmasters, was induced to complete his Undertaking, by the Publication of a Second Volume.

The principal Superiority of the preceding Part of this Work, above other Treatises of the same kind, consists in the numerous Examples and Questions in every Rule, which will enable the Teacher to give different Examples to different Pupils. The same Plan has been continued through the Second Part, which contains the Doctrine of Fractions, both Vulgar and Decimal, with a great variety of Questions, for Illustration and Practice, in all the higher Rules of Arithmetic.

The ensuing Volume contains various Improvements, the Result of actual Experience, and not to be met with in any other Treatise on Arithmetic. The Directions for the Use of Learners were written as occasion required, and are calculated to accelerate the Progress of the Learner, and, at the same time, to lessen the Labour of the Teacher.

The Answers to all the Questions being printed separately, as in the First Part, it is left to the Discretion of the Master, either to communicate them to the Learner or not, as Circumstances may render it expedient or desirable. The Key, it is evident, may occasionally be put into the Hands of the Pupil, with Propriety and Advantage; but when the Answers are invariably annexed to the Questions, as was almost universally the case before the present Treatise appeared, the Learner is prematurely put in possession of that which ought, in general, to be the Result of his own Labour and patient Investigation.

To this Second Part are annexed, a Centenary of Miscellaneous Arithmetical Questions, for the Exercise of Learners, without the Solutions, or even Answers, being printed in the Key. Every teacher can easily supply that Deficiency for himself; and, certainly, to the Pupil, the Questions will always appear more interesting, when it is known that the Answers only exist in the Master's own private Manuscript.

T. M.

Macclesfield,

October 5. 1822.

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PRACTICAL
ARITHMETIC.

PART THE SECOND.

VULGAR FRACTIONS.

Definitions.

A FRACTION, or broken number, is a quantity which expresses a part or parts of a unit, whole, or integer.— A vulgar fraction is the natural or common division of integral numbers into their smaller subdivisions or parts.

The number of parts into which the integer or unit is supposed to be divided, is called the *denominator*, because it denominates the species of parts; and the number of those parts, expressed by the fraction, is called the *numerator*. The method of expressing fractions, or the inferior parts of an integer, is by placing the numerator or number of parts intended by the fraction, above a small line, and the denominator, or number of parts the integer is divided into, below the line. Thus, to express three-fourths of a penny, it is written thus, $\frac{3}{4}$; and five-

eighths of a mile, thus, $\frac{5}{8}$; also eleven-fifteenths, and twenty-seven ninety-fourths of any thing, are written thus, $\frac{11}{15}$ and $\frac{27}{94}$.

The numerator and denominator are in general called the *terms* of a fraction.

A *simple* fraction consists of one numerator and one denominator; as $\frac{1}{4}$, one-fourth, or $\frac{6}{7}$, six-sevenths.

A *compound* fraction is a fraction of a fraction, and consists of two or more simple fractions, having the word *of* placed between them; as $\frac{1}{4}$ of $\frac{5}{6}$, and $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{9}{11}$.

A *proper* fraction is one whose numerator is less than its denominator; as $\frac{5}{6}$.

An *improper* fraction is one whose numerator is either equal to, or greater than its denominator; as $\frac{6}{6}$, $\frac{7}{6}$, and $\frac{47}{40}$. These are all improper fractions.

All proper fractions are less than a unit, or one; and all improper fractions are either equal to, or greater than a unit, according as the numerator is either equal to, or greater than the denominator.

A *mixed* number is a quantity which consists of a whole number and a fraction; as $10\frac{3}{7}$, which signifies ten units, together with three-sevenths of a unit.

If the numerator or the denominator of a fraction, or both of them, be a fraction, or mixed number, the whole is called a *complex* fraction, as, $\frac{4\frac{3}{20}}{19}$, $\frac{4}{19\frac{3}{10}}$, and $\frac{4\frac{3}{20}}{19\frac{3}{10}}$.

Any integer may be converted into an improper fraction, by writing a unit or 1 under it, for a denominator; thus, 12, or 12 units, expressed fraction-wise, will be $\frac{12}{1}$; and 99 will be $\frac{99}{1}$.

The *common measure* of a Vulgar Fraction is that number which will divide both its terms, that is, both its numerator and denominator, without any remainder. Thus, 4 is the common measure of $\frac{12}{10}$, and 9 is the common measure of $\frac{63}{108}$.

If the terms of a simple fraction be divided by the same number, being a common measure, the value of the fraction is not altered; thus, $\frac{20}{80}$ of a pound sterling, $\frac{10}{40}$, $\frac{5}{20}$, or $\frac{1}{4}$, are fractions of the same value,—the first, $\frac{20}{80}$, being equal to twenty three-pences; the second, $\frac{10}{40}$, to ten sixpences; the third, $\frac{5}{20}$, to five shillings; and the last, $\frac{1}{4}$, being one-fourth of a pound, or five shillings.

And hence, if the terms of a simple fraction be multiplied by the same number, its value is still the same: thus, $\frac{1}{4}$ of a pound sterling, $\frac{2}{8}$, $\frac{10}{40}$ or $\frac{20}{80}$, are fractions of the same value,—one-fourth of a pound, or five shillings, being equal to two half-crowns, ten sixpences, or twenty three-pences.

REDUCTION

OF

VULGAR FRACTIONS.

REDUCTION of Vulgar Fractions is the changing of fractional quantities from one form or denomination to another, without altering their values, as preparatory to the operations of Addition, Subtraction, Multiplication, and Division.

CASE I.

To abbreviate or reduce Fractions to their lowest Terms.

RULE. — Divide the numerator and the denominator of the given Fraction by any number which will divide them without a remainder, and the quotients will be the terms of a new fraction, equal in value to the given fraction, but in lower terms. If the same operation be repeated till the terms of the reduced fraction are not divisible by any common measure, greater than 1, the given fraction will then be reduced to its lowest terms.

EXAMPLE FOR ILLUSTRATION.

Reduce $\frac{112}{128}$ to its lowest terms.

$$\frac{112}{128} = \frac{14}{16} = \frac{7}{8}.$$

Divisors, (8) (2)

Here the given fraction is first reduced to $\frac{14}{16}$, by dividing both its terms by 8; and $\frac{14}{16}$ is again reduced to $\frac{7}{8}$, by dividing both the numerator and the denominator by 2. In the same manner, $\frac{126}{105} = \frac{63}{52.5} = \frac{9}{7.5} = \frac{3}{2.5}$

EXAMPLES FOR PRACTICE.

Reduce the following Fractions to their lowest Terms.

1.	$\frac{84}{108}$	=	7.	$\frac{114}{117}$	=
2.	$\frac{72}{136}$	=	8.	$\frac{276}{534}$	=
3.	$\frac{261}{522}$	=	9.	$\frac{120}{10200}$	=
4.	$\frac{144}{324}$	=	10.	$\frac{25200}{36400}$	=
5.	$\frac{180}{354}$	=	11.	$\frac{764}{5240}$	=
6.	$\frac{125}{1000}$	=	12.	$\frac{7845}{96780}$	=

To find the greatest Common Measure of any two given Numbers.

RULE.—Divide the greater number by the less, and the preceding divisor by the remainder, and so on, continually, till nothing remains: the last divisor will be the greatest common measure required.

If the terms of any given fraction be divided by their greatest common measure, it will be reduced to its lowest terms at one operation.

REMARK.—If the greatest common measure of the terms of any fraction be 1, it shews that the fraction is already in its lowest terms.

EXAMPLE FOR ILLUSTRATION.

Required the greatest common measure of 825 and 960.

$$\begin{array}{r}
 825)960(1 \\
 135)825(6 \\
 15)135(9
 \end{array}$$

Hence it appears, that 15 is the last divisor, or the greatest common measure of the two given numbers, 825 and 960.

If the terms of the fraction, $\frac{825}{960}$, be divided by 15, the result, $\frac{55}{64}$, is another fraction of the same value, in its lowest terms. In like manner, the greatest common measure of 196 and 336, is 28; and the fraction, $\frac{196}{336}$, reduced to its lowest terms, by dividing the numerator and denominator by their greatest common measure, 28, (or by its component parts, 4 and 7) will be $\frac{7}{12}$.

EXAMPLES FOR PRACTICE.

13. Reduce $\frac{748}{916}$ to its lowest terms, at one division.
 14. Reduce $\frac{468}{845}$ to its lowest terms.
 15. Reduce $\frac{2548}{2912}$ to its lowest terms.
 16. What is the greatest common measure of 1511 and 8527, being the terms of the fraction $\frac{1511}{8527}$?

CASE II.

To reduce a Fraction to the lowest Terms, when the Numerator and the Denominator consist of two or more Numbers, with the Sign of Multiplication between them.

RULE 1.—If there be any number which is common to both numerator and denominator, cancel it in each.

EXAMPLE FOR ILLUSTRATION.

$$\frac{25 \times 14 \times 8 \times 3}{10 \times 7 \times 14 \times 6} = \frac{25 \times \cancel{14} \times 8 \times 3}{10 \times 7 \times \cancel{14} \times 6}.$$

2. Of the remaining numbers, if there be any two (one in the numerator, and the other in the denominator,) which will divide by some common measure, divide each of them by the common divisor, placing the quotients or results, the one over the number in the numerator, and the other under the number in the denominator.

$$\frac{25 \times 8 \times 3}{10 \times 7 \times 6} = \frac{\overset{5}{\cancel{25}} \times 8 \times 3}{\cancel{10} \times 7 \times \underset{2}{\cancel{6}}}.$$

3. When either of the quotients is a unit or 1, it may be omitted, only cancelling the number which is divided.

$$\frac{5 \times 8 \times 3}{2 \times 7 \times 6} = \frac{5 \times 8 \times \cancel{3}}{2 \times 7 \times \cancel{6}}.$$

4. The quotients or results may frequently be divided by some common measure, in the same manner as the original numbers.

$$\frac{5 \times 8}{2 \times 7 \times 2} = \frac{5 \times \overset{4}{\cancel{8}}}{2 \times 7 \times \cancel{2}}.$$

Lastly, divide 4 in the numerator, and 2 in the denominator, by 2, and the fraction, $\frac{25 \times 14 \times 8 \times 3}{10 \times 7 \times 14 \times 6}$, will be abbreviated or reduced to its lowest terms.

$$\frac{5 \times 4}{2 \times 7} = \frac{5 \times \overset{2}{\cancel{4}}}{\cancel{2} \times 7} = \frac{10}{7}.$$

REM. — This mode of abbreviation is in constant use by all who are acquainted with its value and extensive application. It is particularly convenient, for instance, in the solution of questions in the Rule-of-Three, both Single and Compound, as well as in various other operations, where the dividend and divisor consist of a combination of different numbers, having the sign of multiplication between them. But here it is proper to observe, that a different mode of operation is to be practised, where the numbers have the sign of addition or subtraction between them.

EXAMPLES FOR PRACTICE.

17. Reduce $\frac{3 \times 4 \times 6}{7 \times 3 \times 12}$ to its lowest terms.

18. Reduce $\frac{3 \times 7 \times 48}{72 \times 4 \times 6}$ to its lowest terms.

19. Reduce $\frac{15 \times 4 \times 3}{8 \times 25 \times 16}$ to its lowest terms.

20. Reduce $\frac{5 \times 72 \times 42 \times 10}{27 \times 10 \times 60 \times 105}$ to its lowest terms.

CASE III.

To reduce an Improper Fraction to an equivalent Whole or Mixed Number.

RULE.—Divide the numerator by the denominator, and the quotient, if nothing remains, is an integer or whole number; but if there happens to be a remainder, write it over a small line with the divisor below, for the fractional part of the mixed number.

EXAMPLES FOR ILLUSTRATION

Reduce $\frac{28}{4}$, $\frac{35}{8}$, and $\frac{889}{11}$, to their proper terms.

$$\frac{28}{4} = 7. \quad \frac{35}{8} = 4\frac{3}{8}. \quad \frac{889}{11} = 80\frac{9}{11}.$$

EXAMPLES FOR PRACTICE.

Reduce the following improper fractions to their proper terms.

$$21. \quad \frac{751}{12} =$$

$$22. \quad \frac{927}{91} =$$

$$23. \quad \frac{5421}{22} =$$

$$24. \quad \frac{4483}{35} =$$

$$25. \quad \frac{871871}{1000} =$$

$$26. \quad \frac{801507}{613} =$$

CASE IV.

To reduce a Mixed Number to an equivalent Improper Fraction.

RULE.—Multiply the integral part by the denominator of the fraction, to the product add the numerator of the fraction, and the sum placed above the denominator will form the fraction required.

EXAMPLES FOR ILLUSTRATION.

Reduce $9\frac{5}{17}$, to an improper fraction.

$$9\frac{5}{17} = \frac{5 \quad 9 \times 17 + 5 \quad 158}{17 \quad 17 \quad 17}.$$

In like manner, $7\frac{12}{19} = \frac{145}{19}$.

EXAMPLES FOR PRACTICE.

Reduce the following mixed numbers to their equivalent improper fractions.

27. $9\frac{7}{12} =$ 28. $18\frac{1}{10} =$ 29. $415\frac{11}{16} =$		30. $1000\frac{19}{59} =$ 31. $381\frac{5}{63} =$ 32. $7218\frac{87}{1514} =$
---	--	---

CASE V.

To reduce a Compound Fraction to an equivalent Simple one.

RULE. — Write all the numerators, with the sign of multiplication between them, above a line, and all the denominators, in the same manner, below it. Then abbreviate the numbers as in Case 2, and the result will be the terms of the simple fraction required.

If any number of the given compound fraction be a whole number, (as in Ex. 36) it may be converted into a fraction, by writing 1 for its denominator; and if a mixed number occurs, (as in Ex. 35) it may be reduced to an improper simple fraction by Case 4.

EXAMPLES FOR ILLUSTRATION.

Reduce $\frac{2}{3}$ of $\frac{11}{8}$ of $\frac{6}{7}$ to a simple fraction.

$$\frac{2 \times 11 \times \cancel{6}}{3 \times \cancel{8} \times 7} = \frac{22}{63}$$

Reduce $\frac{4}{5}$ of 6 of $18\frac{5}{6}$ to a simple fraction.

$$\frac{4 \times \cancel{6} \times 113}{5 \times 1 \times \cancel{6}} = \frac{452}{5} = 90\frac{2}{5}.$$

EXAMPLES FOR PRACTICE.

Reduce the following compound fractions to simple fractions of the same value.

33. $\frac{2}{9}$ of $\frac{1}{2}$ of $\frac{5}{11} =$
 34. $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{9}$ of $\frac{14}{17} =$
 35. $\frac{7}{16}$ of $\frac{3}{10}$ of $4\frac{2}{7} =$
 36. $\frac{8}{9}$ of $\frac{7}{24}$ of 36 of $\frac{10}{11} =$
 37. $\frac{5}{8}$ of $\frac{3}{13}$ of $6\frac{8}{9}$ of $2\frac{2}{3} =$

CASE VI.

To find the Value of a Fraction in Numbers of inferior Denomination.

RULE. — Multiply the numerator by the denomination of the given fraction, divide the result by the denominator, and the several quotients as found in that and the other inferior denominations, by compound division, will be the value required.

EXAMPLES FOR ILLUSTRATION.

What is the value of $\frac{5}{7}$ of a pound sterling, or, which is equal thereto, of $\frac{1}{4}$ of 5 pounds?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 7 \overline{) 5 \text{ " } 0 \text{ " } 0} \\ \hline 0 \text{ " } 14 \text{ " } 3 \frac{1}{4} \frac{5}{7}. \end{array}$$

What is the value of $\frac{3}{8}$ of half-a-guinea?

$$\begin{array}{r} \text{s.} \quad \text{d.} \quad \text{£.} \quad \text{s.} \quad \text{d.} \\ 3 \times 10 \text{ " } 6 \quad \quad 1 \text{ " } 11 \text{ " } 6 \quad \quad \text{s.} \quad \text{d.} \\ \hline 8 \quad \quad \quad 8 \quad \quad \quad = 3 \text{ " } 11 \frac{1}{4}. \end{array}$$

EXAMPLES FOR PRACTICE.

38. What is the value of $\frac{7}{8}$ of a sovereign?
39. What is the value of $\frac{3}{8}$ of a guinea?
40. What is the value of $\frac{5}{9}$ of a crown?
41. What is the value of $\frac{4}{5}$ of a shilling?
42. What is the value of $\frac{7}{15}$ of half-a-crown?
43. What is the value, in pounds, &c. of $\frac{1195}{16}$ of a shilling?
44. What is the value of $\frac{5474}{225}$ of a pound?
45. What is the value of $\frac{3}{5}$ of a lb. avoirdupois?
46. What is the value of $\frac{5}{14}$ of 1 cwt?
47. What is the value of $\frac{4}{7}$ of a mile?
48. What is the value of $\frac{5}{8}$ of an acre?
49. What is the value of $\frac{4}{15}$ of a barrel of ale or beer?
50. Required the value, in days, hours, and minutes, of $\frac{4}{31}$ of a month.

CASE VII.

To reduce a given Fraction, or any Quantity, from one Denomination to any other assigned Denomination.

RULE. — When the reduction is from a greater to a less denomination, and consequently is performed by multiplication, — annex the proper multiplier, or mul-

Occasional Directions for the Learner.

42. It is more convenient, in this example, to divide the value or denomination of the integer (2s. 6d.) by the denominator of the fraction, and to multiply the quotient by the numerator.

43. Divide 1195s. by the component parts of 16, and reduce the shillings in the quotient to pounds.

44. Divide £5474 " 0 " 0 by the three component parts of 225, and the last quotient will be the required value.

multipliers, to the numerator of the given fraction, with the sign of multiplication between them. On the contrary, when the reduction is from a less denomination to a greater, and consequently is performed by division, — annex the proper divisor, or divisors, to the denominator of the given fraction, with the sign of multiplication between them. When the process of reduction involves both multiplication and division, — annex the multipliers to the numerator, and the divisors to the denominator of the given fraction, with the sign of multiplication between each. Having made these preparatory arrangements, as the circumstances may require, — reduce the whole, by Case II. to a simple fraction, in its lowest terms, for the answer.

REM. If the given quantity be a whole number or integer, write 1 under it, for a denominator. If it be a mixed number, reduce it to an improper fraction: and if it be a compound number of two or more denominations, reduce it to the lowest denomination mentioned. But if the given quantity consist of only two denominations, the value of the lower denomination may be annexed, as a fractional quantity, to the number of the higher denomination, and the whole reduced to an improper fraction by Case 4. Thus, 7s. 3d. is equal to $7\frac{1}{4}$ s., or $\frac{29}{4}$ s.

EXAMPLES FOR ILLUSTRATION.

Reduce $\frac{2}{9}$ of a pound to the fraction of a penny.

$$\pounds \frac{2}{9} = \frac{2 \times 20 \times 4}{9 \times 3} = \frac{160}{3} \text{ of a penny.}$$

Proof.

$\begin{array}{r} \pounds. \quad s. \quad d. \\ 9 \overline{) 2 \text{ " } 0 \text{ " } 0} \\ \hline 0 \text{ " } 4 \text{ " } 5 \frac{1}{4} \frac{1}{3} \end{array}$	$\begin{array}{r} d. \\ 3 \overline{) 160} \\ \hline 53 \frac{1}{3}, \text{ or } 4s. \ 5 \frac{1}{4}d. \ \frac{1}{3}q. \end{array}$
---	---

Reduce $\frac{7}{9}$ of a pound to the fraction of a guinea.

$$\pounds \frac{7}{9} = \frac{7 \times 20}{9 \times \frac{7}{3}} = \frac{20}{27} \text{ of a guinea.}$$

Proof.

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 9) 7 \text{ " } 0 \text{ " } 0 \\ \hline 0 \text{ " } 15 \text{ " } 6 \frac{1}{2} \frac{2}{3} . \end{array}$$

$$\begin{array}{r} \text{G.} \quad \text{s.} \quad \text{d.} \\ 9) 20 \text{ " } 0 \text{ " } 0 \\ \hline 3) 2 \text{ " } 4 \text{ " } 8 \\ \hline 0 \text{ " } 15 \text{ " } 6 \frac{1}{2} \frac{2}{3} . \end{array}$$

EXAMPLES FOR PRACTICE.

51. Reduce $\frac{2}{15}$ of a pound to the fraction of a shilling.
52. Reduce $\frac{8}{3}$ of a shilling to the fraction of a pound.
53. Reduce $\frac{1}{2}\frac{1}{8}$ of a pound to the fraction of a farthing.
54. Reduce $\frac{1600}{7}$ of a halfpenny to the fraction of a pound.
55. Reduce 15 shillings (or $1\frac{5}{1}$ s.) to the fraction of a pound.
56. Reduce 11s. 4d. (or $11\frac{1}{3}$ s.) to the fraction of a pound.
57. Reduce $3\frac{7}{40}$ d. to the fraction of a shilling.
58. Reduce $\frac{5}{21}$ of a cwt. to the fraction of a lb.
59. Reduce $29\frac{3}{4}$ lb. to the fraction of a ton.
60. Reduce $13\frac{1}{2}$ bushels to the fraction of a chaldron.
61. Reduce $3323\frac{1}{13}$ seconds to the fraction of a day.
62. Reduce 1 rd. 30 p. to the fraction of an acre.
63. Reduce $\frac{2}{9}$ of a guinea to the fraction of a pound.
64. Reduce $\frac{5}{8}$ of half-a-guinea to the fraction of a shilling.
65. Reduce $6\frac{9}{16}$ shillings to the fraction of half-a-crown.
66. Reduce $2\frac{5}{8}$ half-crowns to the fraction of half-a-guinea.
67. Reduce $\frac{17}{40}$ of an English ell to the fraction of a yard.

CASE VIII.

To reduce a given Complex Fraction to an equivalent Simple one.

When only one of the terms is a mixed number, — multiply both the numerator and denominator of the given fraction by the denominator of the fractional part, and the products will be the terms of the simple fraction required, having only one number in the numerator, and one in the denominator.

REM. — The still more complex Case, when both the terms are fractional or mixed numbers, may be referred to Division of Vulgar Fractions, making the numerator of the fraction the dividend, and the denominator the divisor.

EXAMPLE FOR ILLUSTRATION.

Reduce $\frac{3\frac{2}{5}}{4}$ to an equivalent simple fraction.

$$\frac{3\frac{2}{5} \times 5}{4 \times 5} = \frac{17}{20}.$$

EXAMPLES FOR PRACTICE.

68. Reduce $\frac{4\frac{11}{15}}{19}$ to a simple fraction.

69. Reduce $\frac{4}{19\frac{7}{9}}$ to a simple fraction.

70. Reduce $\frac{17\frac{1}{2}}{100}$ to a simple fraction.

71. Reduce $\frac{5\frac{3}{7}}{8}$ to a simple fraction.

CASE IX.

To reduce any given number of Fractions to a Common Denominator.

This is performed by multiplying the terms of each fraction by an appropriate multiplier. These multipliers are found as follows:

1. When there are only two fractions, the denominator of the second is the multiplier for the first fraction, and the denominator of the first is, in like manner, the multiplier for the second fraction.

2. When there are three fractions, the multiplier for each fraction is the product of the denominators of the other two fractions.

3. When there are more than three fractions, the multiplier for any particular fraction is found by multiplying all the denominators, excepting its own, continually together.

The respective multipliers for each individual fraction, ascertained as above, being afterwards divided by their greatest common measure, will give the answers in their least common denomination.

EXAMPLES FOR ILLUSTRATION.

1. Reduce $\frac{3}{5}$ and $\frac{6}{13}$ to a common denominator.

Given fractions, $\frac{3}{5}$, and $\frac{6}{13}$.

Multipliers, 13 5

Answers, $\frac{39}{65}$, and $\frac{30}{65}$.

2. Reduce $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{3}{7}$, to a common denominator.

Given fractions, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{3}{7}$.

Multipliers, 35 21 15

Answers, $\frac{70}{105}$, $\frac{84}{105}$, and $\frac{45}{105}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{4}{7}$, to equivalent fractions, which shall have a common denominator.

Given fractions, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{4}{7}$.

Multipliers, 105 70 42 30.

Answers, $\frac{105}{210}$, $\frac{140}{210}$, $\frac{126}{210}$, and $\frac{120}{210}$.

4. Reduce $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{11}{20}$ to a common denominator.

Given fractions,	$\frac{5}{6}$,	$\frac{7}{10}$,	and	$\frac{11}{20}$.
Multipliers,	200	120		60
Ditto, reduced,	10	6		3
Answers,	$\frac{50}{60}$,	$\frac{42}{60}$,	and	$\frac{33}{60}$.

5. Reduce $\frac{2}{5}$, $4\frac{1}{6}$, and $\frac{3}{4}$ to a common denominator.

Given fractions, $\frac{2}{5}$, $\frac{25}{6}$, and $\frac{3}{4}$.

The denominators of these fractions being each aliquot parts of the number 60, the multipliers may be found by dividing that number by the denominator of each fraction respectively.

Multipliers,	12	10		15
Answers,	$\frac{24}{60}$,	$\frac{250}{60}$,	and	$\frac{45}{60}$.

EXAMPLES FOR PRACTICE.

72. Reduce $2\frac{1}{3}$, $\frac{5}{9}$, and $\frac{3}{4}$ to a common denominator.
 73. Reduce $\frac{3}{5}$, $\frac{1}{3}$ of $\frac{4}{5}$, and $3\frac{9}{10}$ to a common denominator.
 74. Reduce $\frac{2}{7}$, $\frac{3}{5}$, and $5\frac{1}{6}$ to a common denominator.
 75. Reduce $\frac{4}{15}$, and $\frac{3}{20}$ to their least common denominator.
 76. Reduce $\frac{5}{21}$, and $\frac{9}{14}$ to a common denominator.
 77. Reduce $\frac{9}{10}$ of $6\frac{7}{8}$, $\frac{4}{7}$, $\frac{2}{3}$, and $7\frac{1}{2}$ to a common denominator.

REM. The proper multipliers for the fractions in each of the following examples, may be easily determined, by comparing their denominators with certain common multiples, of which they are respectively aliquot parts.

78. Reduce $\frac{7}{9}$, and $\frac{5}{36}$ to a common denominator.

Occasional Directions for the Learner.

78. As 9, the denominator of the first fraction, is a fourth part of 36, if the terms of that fraction be multiplied by 4, and those of the second by 1, the fractions will be reduced to a common denominator, without changing the terms of the second fraction.

79. Reduce $\frac{2}{5}$, $\frac{3}{20}$, and $\frac{7}{12}$ to a common denominator.
80. Reduce $\frac{5}{16}$, $7\frac{1}{2}$, and $5\frac{3}{8}$ to a common denominator.
81. Reduce $\frac{1}{4}$, $1\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to a common denominator.
82. Reduce $5\frac{1}{6}$, $\frac{4}{15}$, and 7 to a common denominator.

CASE X.

To find the least Common Multiple of any Series of Numbers, being the Denominators of any given Number of Fractions, and thereby to reduce those Fractions to a Common Denominator in the least Terms.

1. Write the given numbers, or denominators, with a dot or period between each number, in a straight line.

2. Divide any two or more of these numbers by some common measure, (placed as a divisor, on the left of the numbers,) and write each quotient under its respective number, with the undivided numbers, as before, in the same line.

3. Divide any two or more numbers in this new series, by some common measure, placing the common measure or divisor to the left, and the several quotients and undivided numbers below, as in the preceding part.

4. Repeat the same operation again, if the numbers will admit of it, till there are no two numbers left, which can be divided by any common measure, greater than 1, without a remainder.

5. Multiply the several divisors, and the numbers in

Occasional Directions for the Learner.

79. The denominators of the fractions, in this Example, are each aliquot parts of 60.

the last line, continually together, and the product will be the least common multiple of the original or given series of numbers.

6. Divide the least common multiple by the denominators of each fraction respectively, and the several quotients will be the multipliers required, for reducing the given fractions to their least common denominator.

EXAMPLE FOR ILLUSTRATION.

Reduce $\frac{2}{3}$, $\frac{1}{5}$, $\frac{7}{8}$, and $\frac{3}{10}$ to equivalent fractions, having a common denominator in the least terms.

5		3	.	5	.	8	.	10
2		3	.	1	.	8	.	2
		3	.	1	.	4	.	1

Then, $5 \times 2 \times 3 \times 1 \times 4 \times 1 = 120$, the least common multiple of 3, 5, 8, and 10.

$\frac{120}{3} = 40$, the first multiplier. $\frac{120}{5} = 24$, the second multiplier.

$\frac{120}{8} = 15$, the third multiplier. $\frac{120}{10} = 12$, the fourth or last multiplier.

Given fractions,	$\frac{2}{3}$,	$\frac{1}{5}$,	$\frac{7}{8}$,	and	$\frac{3}{10}$.
Multipliers,	40	24	15		12
Answers,	$\frac{80}{120}$,	$\frac{24}{120}$,	$\frac{105}{120}$,	and	$\frac{36}{120}$.

EXAMPLES FOR PRACTICE.

83. Reduce $\frac{4}{11}$, $\frac{5}{6}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator in the least terms.

84. Reduce $\frac{5}{8}$, $\frac{3}{4}$ of $\frac{2}{5}$, $3\frac{7}{10}$, and $\frac{1}{2}\frac{9}{4}$ to a common denominator.

QUESTIONS FOR EXAMINATION.

What is a Fraction?

What are the Terms of a Vulgar Fraction?

Which is the Numerator, and which the Denominator?

What does the Denominator shew ?

What does the Numerator express ?

Why are these Fractions denominated Vulgar Fractions ?

What is meant by a Simple Fraction ? — Give an example.

What is a Compound Fraction, and how is it distinguished ? — Give an example.

What is a Proper Fraction ?

What is an Improper Fraction ? — Give examples of each.

What is a Mixed Number ? — Give an example.

What is a Complex Fraction ? — Give examples of the three different kinds of Complex Fractions.

How may an Integer be converted into a Fractional quantity ?

What is the Common Measure of the Terms of any Fraction ?

What is meant by the Reduction of Vulgar Fractions ?

* * * Similar Questions may be very easily adapted to any of the subsequent Rules ; but as all such Questions are used, according to the discretion of the Master, for the occasional examination of the Pupil, and to impress the Definitions and Rules more strongly on his memory, it was thought sufficient, for the present purpose, to insert the above, merely as a specimen.

ADDITION

OF

VULGAR FRACTIONS.

RULE. — Convert mixed numbers, if any are given, to improper fractions, and compound fractions to simple ones, and reduce them all to their least common denominator. Then add all the numerators together, and write the sum over the common denominator, and it will express the amount of the fractions required; which amount, if it be an improper fraction, must then be reduced to a mixed number, or an integer.

REM. 1. — The sum of mixed numbers is often found without reducing them to improper fractions, by adding the fractions together, and carrying the units in the sum to the integral parts.

2. The denominators of fractions are never added together.

3. Fractions of different integers are usually added together, by finding their respective values, by Case 6, in Reduction, and the sum of those values will be the answer required.

EXAMPLES FOR ILLUSTRATION.

1. What is the sum of $10\frac{2}{3}$, $\frac{4}{9}$ of $\frac{3}{10}$, and $\frac{7}{8}$?

First, $10\frac{2}{3} = \frac{32}{3}$; and $\frac{4}{9}$ of $\frac{3}{10} = \frac{2}{15}$

Given fractions, $\frac{32}{3}$, $\frac{2}{15}$, $\frac{7}{8}$.

Multipliers, 40 8 15

Then, $\frac{1280}{120} + \frac{16}{120} + \frac{105}{120} = \frac{1401}{120} = \frac{467}{40} = 11\frac{27}{40}$, the sum required.

2. What is the sum of $\frac{5}{9}$ of a pound, $\frac{2}{3}$ of half-a-guinea, and $\frac{4}{7}$ of a shilling?

	s.	d.
$\frac{5}{9}$ of a pound =	11	" 1 $\frac{1}{4}$ $\frac{1}{5}$
$\frac{2}{3}$ of half-a-guinea =	6	" 3 $\frac{1}{2}$ $\frac{2}{5}$
$\frac{4}{7}$ of a shilling =	0	" 6 $\frac{3}{4}$ $\frac{3}{7}$ *
The amount,	17	" 11 $\frac{3}{4}$ $\frac{17}{105}$

$$* \frac{1}{3} + \frac{2}{3} + \frac{3}{7} = \frac{35}{105} + \frac{42}{105} + \frac{122}{105} = 1\frac{17}{105} q.$$

EXAMPLES FOR PRACTICE.

1. What is the sum of $\frac{4}{9}$ and $\frac{7}{9}$?
2. What is the sum of $\frac{3}{5}$ and $\frac{2}{15}$?
3. Add together $\frac{2}{7}$, $\frac{5}{8}$, and $11\frac{3}{4}$.
4. What is the sum of $\frac{4}{5}$, $\frac{2}{5}$ of $\frac{1}{3}$, and $9\frac{3}{10}$?
5. What is the sum of $6\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{7}$?
6. What is the sum of $\frac{4}{7}$ of $2\frac{1}{2}$, $14\frac{1}{3}$, $\frac{9}{10}$, and $\frac{7}{8}$?
7. If I have $\frac{3}{8}$ of a ship, and purchase another share of $\frac{5}{16}$; what part of her will then belong to me?
8. What is the sum of $\frac{2}{9}$ of a pound, $\frac{1}{7}$ of a shilling, and $\frac{7}{12}$ of a penny?
9. What is the sum of $\frac{1}{5}$ of a guinea, $4\frac{4}{7}$ pounds, and $\frac{3}{5}$ of $\frac{5}{7}$ of a shilling?
10. What is the sum of $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a foot?
11. Add together $\frac{2}{3}$ of a week, $\frac{3}{4}$ of a day, and $\frac{4}{15}$ of an hour?
12. What is the sum of $\frac{2}{5}$ of a yard, and $\frac{3}{7}$ of a foot?

SUBTRACTION

OF

VULGAR FRACTIONS.

RULE. — Reduce the given fractions to the same denominator, as in the preceding rule; then subtract the

Occasional Directions for the Learner.

11. Here, the two given fractions, having the same denominator, require no preparative reduction; but the sum of the given numerators, written above the common denominator, will be the amount required. — When the answer is an improper fraction, it must be reduced to a whole or mixed number, by Case 3.

less numerator from the greater, and the remainder written over the common denominator, will be the difference required.

REM. — To subtract fractions of different integers; find their respective values, and then proceed as in compound subtraction. — When mixed numbers are to be subtracted, without reducing them to improper fractions, the fractional parts must be brought to a common denominator, and the numerator of the lower fraction subtracted from that of the upper. But if the lower fraction be greater than the upper, subtract its numerator from the common denominator (which is always the numerator of a fraction equal to a unit), to the remainder add the numerator of the upper fraction, and, in that case, carry one to the units' figure of the lower whole number.

EXAMPLES FOR ILLUSTRATION.

From $12\frac{1}{3}$ subtract $\frac{5}{9}$.

$$\begin{array}{r} 73 \\ 6 \end{array} - \frac{5}{9}$$

Multipliers, $3 \quad 2$

$$\frac{219}{18} - \frac{10}{18} = \frac{209}{18} = 11\frac{11}{18}.$$

From	$9\frac{7}{12}$	$9\frac{1}{6}$	9
Take	$3\frac{1}{6}$	$3\frac{7}{12}$	$3\frac{7}{12}$
Remainders,	<u>$6\frac{5}{12}$</u>	<u>$5\frac{7}{12}$</u>	<u>$5\frac{5}{12}$</u>

EXAMPLES FOR PRACTICE.

1. From $11\frac{1}{2}$ take $\frac{3}{16}$.
2. From $\frac{17}{2}$ take $\frac{1}{7}$.
3. From $5\frac{3}{8}$ subtract $\frac{2}{7}$ of $6\frac{1}{4}$.
4. From $96\frac{3}{7}$ take $14\frac{1}{3}$.
5. From $14\frac{1}{2}$ take $\frac{2}{3}$ of 19.
6. From $1\frac{5}{14}$ take $\frac{4}{7}$ of $\frac{1}{6}$ of 7.
7. Suppose I had $\frac{5}{8}$ of a ship, and that I sold $\frac{2}{3}$ of my share; what part of her have I left?
8. From $\frac{7}{8}$ of a moidore subtract $\frac{9}{16}$ of a guinea.
9. What is the difference between the full weight of a

half-guinea and that of a shilling, the weight of the former being 2 dwts. $16\frac{6}{8}\frac{4}{9}$ gr., and that of the latter 3 dwts. $20\frac{2}{3}\frac{8}{1}$ gr.

10. Express the difference, in troy-weight, between the ounce avoirdupois and the ounce troy, 1 oz. avoirdupois being equal to 18 dwts. $5\frac{1}{3}\frac{5}{2}$ gr. troy.

11. From $7\frac{7}{10}$ weeks take $2\frac{4}{5}$ hours.

MULTIPLICATION

OF

VULGAR FRACTIONS.

RULE. — Reduce mixed numbers, if any are given, to improper fractions, and compound fractions to simple ones; then write all the numerators, with the sign of multiplication between them, above a line, and all the denominators, in the same manner, below it. Abbreviate the numbers, as in Reduction, Case 2, and the last result will be the product required.

EXAMPLE FOR ILLUSTRATION.

What is the continual product of $2\frac{1}{5}$, $\frac{2}{11}$ of $\frac{5}{6}$, and $\frac{9}{10}$?

First, $2\frac{1}{5} = \frac{11}{5}$; and $\frac{2}{11}$ of $\frac{5}{6} = \frac{5}{33}$.

$$\text{Then, per rule, } \frac{\overset{3}{11} \times \cancel{8} \times \cancel{9}}{\cancel{8} \times \cancel{33} \times 16} = \frac{3}{16}, \text{ the product.}$$

EXAMPLES FOR PRACTICE.

1. Multiply $\frac{3}{4}$ by $\frac{5}{6}$.
2. Multiply $\frac{5}{6}$ of $\frac{7}{8}$ by $3\frac{3}{4}$.

3. Multiply $\frac{4}{14}$, $18\frac{4}{5}$, and $\frac{3}{4}$ of $\frac{3}{7}$ of $\frac{1}{3}$ continually together.

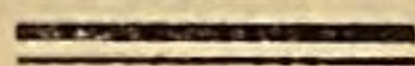
4. Required the continual product of $4\frac{1}{2}$, $3\frac{1}{4}$, and $\frac{2}{9}$ of $4\frac{1}{6}$.

5. Multiply 6, $\frac{2}{9}$ of $\frac{3}{5}$, and $\frac{5}{8}$ of $\frac{2}{7}$ continually together.

6. What is the product of $\frac{3}{7}$ of $\frac{4}{9}$, $\frac{1}{2}$ of $\frac{14}{15}$, and $9\frac{2}{7}$?

7. From $\frac{3}{8}$ subtract $\frac{1}{3}$ of $\frac{5}{6}$, and multiply the remainder by $73\frac{1}{2}$.

8. How many yards are contained in $3\frac{1}{2}$ pieces of silk, each $24\frac{1}{3}$ yards.



DIVISION

OF

VULGAR FRACTIONS.

RULE. — Prepare the fractions, as in the preceding rule, then multiply the terms of the dividend by the reciprocal of the divisor, and the product will be the quotient required.

When a fraction is to be divided by a whole number, it is performed by annexing the whole number to the denominator of the fraction, with the sign of multiplication between them, and then reducing the whole to a simple fraction.

REM. — The reciprocal of a fraction is got by inverting its terms, writing the denominator in the numerator's place, and the numerator in the denominator's place; thus, the reciprocal of $\frac{6}{7}$ is $\frac{7}{6}$, and the reciprocal of 5, or $\frac{5}{1}$, is $\frac{1}{5}$.

EXAMPLE FOR ILLUSTRATION.

Divide $\frac{14}{9}$ by $\frac{7}{13}$.

The reciprocal of $\frac{7}{13}$ is $\frac{13}{7}$

Then, *per rule*, $\frac{\overset{2}{14} \times \overset{5}{13}}{\underset{3}{9} \times 7} = \frac{10}{3}$, or $3\frac{1}{3}$, the quotient.

Divide $\frac{9}{16}$ by 6.

$\frac{\overset{3}{9}}{\underset{2}{16} \times 6} = \frac{3}{32}$, the quotient.

EXAMPLES FOR PRACTICE.

1. Divide $\frac{16}{25}$ by $\frac{3}{4}$.
2. Divide $\frac{5}{6}$ by $2\frac{1}{7}$.
3. Divide $\frac{12}{35}$ by $\frac{2}{7}$.
4. Divide $1\frac{5}{14}$ by $1\frac{1}{6}$.
5. Divide $9\frac{5}{9}$ by $7\frac{1}{3}$.
6. Divide $\frac{5}{7}$ of $7\frac{3}{5}$ by $\frac{2}{3}$ of $\frac{1}{3}$.
7. Divide 25 by $14\frac{7}{12}$.
8. Divide $14\frac{7}{12}$ by 25.
9. Divide $485\frac{11}{32}$ by $13\frac{11}{12}$.
10. From 3 subtract $\frac{7}{8}$ of $\frac{2}{7}$ of $\frac{5}{9}$, and divide the remainder by $7\frac{5}{12}$.

THE RULE-OF-THREE DIRECT,

IN

VULGAR FRACTIONS.

RULE.— State the question, as in whole numbers. Reduce mixed numbers to improper fractions, compound

fractions to simple ones, and the first and third terms to the same denomination. Then multiply together the reciprocal of the first term, and the second and third terms, and the result will be the answer to the question, in the same denomination as the middle term.

QUESTION FOR ILLUSTRATION.

If $4\frac{1}{5}$ yards cost £4 " 1 " 8; what will $3\frac{1}{2}$ yards cost?

Stated thus,

$$\begin{array}{ccccc} \text{Yds.} & & \text{£. s. d.} & & \text{Yds.} \\ \text{If } 4\frac{1}{5} & \text{-----} & 4 \text{ " } 1 \text{ " } 8 & \text{-----} & 3\frac{1}{2} \end{array}$$

The first and third terms being reduced to improper fractions, and the second term to the fraction of a pound, will stand thus:

$$\begin{array}{ccccc} \text{Yds.} & & \text{£.} & \text{£.} & \text{Yds.} \\ \text{If } \frac{21}{5} & \text{-----} & 4\frac{1}{12} & = & \frac{49}{12} \text{-----} \frac{7}{2} \end{array}$$

$$\text{Then, } \frac{5 \times 49 \times 7}{21 \times 12 \times 2} = \frac{\text{£}245}{72} = \text{£}3 \text{ " } 8 \text{ " } 0\frac{1}{2} \frac{2}{3}, \text{ the answer.}$$

$$\begin{array}{r} \text{£. s. d.} \\ 8 \left| \begin{array}{l} 245 \text{ " } 0 \text{ " } 0 \\ 30 \text{ " } 12 \text{ " } 6 \\ \hline 3 \text{ " } 8 \text{ " } 0\frac{1}{2} \frac{2}{3} \end{array} \right. \text{ the value of } \text{£}\frac{245}{72}. \end{array}$$

QUESTIONS FOR PRACTICE.

1. If $\frac{3}{16}$ of a yard of velvet cost $\frac{1}{5}$ of a pound; what will $\frac{5}{8}$ of a yard cost?
2. What will $3\frac{7}{16}$ oz. of silver cost, at 5s. 4d. an ounce?
3. If $\frac{3}{7}$ of a ship be worth £250 " 10 " 0; what is $\frac{1}{8}$ of her worth?
4. What is the interest of £458 " 15 " 0, at the rate of $\text{£}4\frac{1}{4}$ interest per cent.?
5. Required the commission of £730 " 2 " 8, at 3s. 4d. commission per cent.
6. What will $18\frac{2}{3}$ lb. cost, at the rate of £13 " 2 " 6 per cwt.?
7. If $1\frac{1}{2}$ dollar be worth $7\frac{3}{40}$ s.; what is the value of 500 dollars?

8. If $\frac{7}{8}$ cwt. cost 14 guineas; what will 7 cwt. 2 qrs. cost?

9. If $\frac{3}{5}$ of a yard cost $\frac{2}{3}$ of 19s.; what will $5\frac{1}{5}$ yards cost?

10. What will 4 pieces, each containing $23\frac{7}{8}$ ells, come to, at 3s. $1\frac{1}{2}d.$ per ell?

11. If $\frac{5}{32}$ of a ship be worth £227.12.1; what part of her may be bought for £273.2.6?

12. How much Bank-stock may be bought for £1336.1.9, at £108 $\frac{5}{8}$ per cent. Bank-stock?

13. What must be given for £2500 India stock, at £162 $\frac{3}{8}$ per cent. India-stock?

14. How much South-sea-stock, at £111 $\frac{3}{8}$ per cent. South-sea-stock, will £50 000 purchase?

15. What is the insurance of a ship and cargo, value £2700.10.0, at £8.8.0 insurance per cent.?

16. What is the value of $11\frac{1}{4}$ pieces of silk, each piece containing $24\frac{3}{8}$ yards, at 6s. $0\frac{1}{4}d.$ per yard?

Occasional Directions for the Learner.

13. Proof. Multiply £162.7.6 by the component parts of 25, and the product will be the answer.

14. The answer to this question is £40 000 000 $\div$ 891; and the component parts of the divisor (891) are 9, 9, and 11; or, 11, 9, and 9; or, 9, 11, and 9; or, by way of proof, 11, 3, 9, and 3. — In compound division, as in the present instance, it frequently happens that there is a fraction at the end of the dividend, and if there be no remainder, the fractional part alone is divided, by annexing the divisor to the denominator of the fraction.

Thus, $\frac{3}{5} q.$ divided by 4, is $\frac{3}{5 \times 4} q. = \frac{3}{20} q.$ the quotient.

But when there is a remainder to be carried to the fractional part, the rule is, multiply the remainder by the denominator of the fraction, to the product add the numerator of the fraction, for a new numerator, and annex the divisor, as before, to the denominator. Thus, suppose the fraction at the end of the dividend to be $\frac{3}{5} q.$ the remainder 7q.,

and the divisor 8; the result, in this case, will be $\frac{19}{5 \times 8} q. = \frac{19}{40} q.$

THE RULE-OF-THREE INVERSE,

IN

VULGAR FRACTIONS.

RULE. — Prepare the terms, and state the question, as in the Rule-of-Three direct. Then multiply the first and second terms by the reciprocal of the third term, and the result will be the answer, in the same denomination as the middle term.

QUESTION FOR ILLUSTRATION.

What is wheat per bushel, when the penny loaf weighs 5 oz. 10 dr., if it weigh 9 ounces, when wheat is 4s. 2d. per bushel?

Preparation of the Terms.

First, 4s. 2d. = $4\frac{1}{6}$ s. = $\frac{25}{6}$ s. the middle term, and 9 oz. = $\frac{9}{1}$ oz., the other term of supposition; also, 5 oz. 10 dr. = $5\frac{5}{8}$ oz. = $\frac{45}{8}$ oz., the term of demand.

The Stating.

$$\text{If } \frac{9}{1} \text{ oz.} \quad \frac{25}{6} \text{ s.} \quad \frac{45}{8} \text{ oz.} ?$$

Operation.

$$\frac{9 \times 25 \times 8}{1 \times 6 \times 45} = \frac{20}{3} \text{ s.} = 6\text{s. } 8\text{d., the answer}$$

$$3) 20\text{s.}$$

$$\underline{\underline{6\text{s. } 8\text{d., the value of } \frac{20}{3}\text{s.}}}$$

REM. The only difference, then, in the operation, between the Rule-of-Three Direct, and the Rule-of-Three Inverse, is, that in the former you use the reciprocal of the first term, and, in the latter, the reciprocal of the third term.

QUESTIONS FOR PRACTICE.

1. If 18 men finish a piece of work in $31\frac{1}{4}$ days; in how many days will 8 men do the same?
2. What quantity of shalloon that is $\frac{7}{8}$ yd. wide, will line $9\frac{1}{2}$ yards of cloth which is $2\frac{1}{4}$ yds. wide?
3. How many pieces of cloth, at two guineas per piece, are equal in value to $120\frac{3}{8}$ pieces, at $23\frac{5}{8}$ s. per piece?
4. If in $17\frac{2}{5}$ days, 20 persons spend a certain sum of money; in how many days will 9 persons spend the same?
5. How many lbs. of tea, at 8s. $7\frac{1}{2}$ d. per lb. must be given for $5\frac{8}{9}$ cwt. of sugar, at $6\frac{3}{4}$ d. per lb.?
6. How many ells (English) of cloth that is 7 quarters wide, will be sufficient to line 12 yards of cloth which is half-ell wide?

THE RULE-OF-FIVE,

IN

VULGAR FRACTIONS.

RULE. — State the question, as in whole numbers, and reduce the terms, as in the Single Rule-of-Three. Then multiply the terms of the dividend and the reciprocals of the divisors continually together, and the result will be the answer to the question, in the same denomination as the middle term.

QUESTION FOR ILLUSTRATION.

What principal will clear £38 " 10 " 0 interest, in a year and a quarter, at £3 " 10 " 0 per cent. per annum?

The Stating.

£.		£.		£.	
* $3\frac{1}{2}$	—	100	—	$38\frac{1}{2}$	(Direct.)
Yr.				Yr.	
1	—	×	—	* $1\frac{1}{4}$	(Inverse.)
		c	3		

Reduction of the terms.

$$\begin{array}{ccccc} * \frac{7}{2} & \text{---} & \frac{100}{1} & \text{---} & \frac{77}{2} \\ \frac{1}{1} & \text{---} & \times & \text{---} & * \frac{5}{4} \end{array}$$

Operation.

$$\begin{array}{r} 20 \quad 11 \\ 2 \times 4 \times 100 \times 77 \\ \hline 7 \times 5 \times 1 \times 2 \end{array} = £880, \text{ the answer.}$$

The truth of this answer may be proved by changing the terms of the question, as follows;

What is the interest of £880, for $1\frac{1}{4}$ year, at $£3\frac{1}{2}$ per cent. per annum?

QUESTIONS FOR PRACTICE.

1. If a family of 20 persons spend $£100\frac{7}{9}$, in 30 days; how much will serve a family of 9 persons, $22\frac{1}{2}$ days?

2. If 2 men can do $12\frac{3}{4}$ rods of ditching, in $6\frac{1}{2}$ days; how many men can do $247\frac{2}{13}$ rods, in 14 days?

3. If the carriage of 7 cwt. 21 lb., 64 miles, cost $£1\text{ }7\text{ }4$; what must be paid for the carriage of 5 cwt. 3 qrs., 150 miles?

4. A man and his wife having received, for one day's labour, $3\text{s. } 10\frac{1}{2}\text{d.}$; how much must they have, for $10\frac{1}{2}$ days, their two sons helping them?

5. If 3 seamen, for $9\frac{1}{4}$ months' service, receive $£40\frac{1}{5}$; how much must 100 seamen receive for $28\frac{3}{7}$ months?

6. What is the interest of £325, for 15 months, at $£4\frac{1}{4}$ per cent. per annum?

7. If I pay $16\text{s. } 4\text{d.}$ for the carriage of $5\frac{1}{4}$ cwt., 20 miles; what weight may I have carried $7\frac{1}{2}$ miles, for $£1\text{ }0\text{ }8\frac{1}{2}$?

8. What principal, being put to interest for 9 months, will gain 50 guineas, at $£4\text{ }10\text{ }0$ per cent. per annum?

9. Required the interest of $£1555\text{ }11\text{ }1\frac{1}{4}\text{ } \frac{1}{3}$, for 9 months, at $£4\frac{1}{2}$ per cent. per annum.

MISCELLANEOUS QUESTIONS,

For the Learner's Exercise.

1. Express 45 by four fours, and 1000 by nine nines.
2. From 1 subtract $\frac{3}{10}$ of $\frac{18}{7}$ of $\frac{141}{213}$.
3. From 3 subtract $\frac{1}{16}$ of $\frac{47}{19}$, and to the remainder add $\frac{1}{25}$ of $\frac{7}{8}$.
4. To 8 add $\frac{1}{3}$ of $7\frac{1}{2}$ minus $\frac{29}{30}$ of $1\frac{1}{4}$, and from the sum subtract $\frac{1}{11}$ of 12 plus $\frac{1}{19}$ of 27.
5. Multiply $9\frac{28}{37}$ by $\frac{1}{3}$ of $\frac{3}{16}$, and find the square of the product.
6. Divide 1 by $\frac{3}{4}$ of $\frac{7}{8}$ of $2\frac{3}{4}$, and find the cube or third power of the quotient.
7. How many pounds are there in $18\frac{1}{2}$ cheeses, each weighing, on an average, $115\frac{3}{4}$ lb.?
8. What number multiplied by $\frac{5}{12}$ of 3 will produce $56\frac{1}{2}$?
9. What is each person's share of a prize of £30 000, one-sixteenth of which is to be divided among 13 persons?
10. A person, who was possessed of $\frac{3}{5}$ of a copper-mine, sold $\frac{3}{4}$ of his share for £2 280; what was the value of the whole mine at the same rate?
11. A person making his will, gave to one child $\frac{19}{30}$ of his estate, to another $\frac{11}{9}$, and when these legacies came to be paid, one turned out £540, 10, 0 more than the other; what did the testator die worth?
12. A gentleman having a certain number of crowns said, if $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{6}$ of what he had, were added together, they would make 45; how many crowns had he?
13. A father devised $\frac{34}{83}$ of his estate to one of his sons,

Occasional Directions for the Learner.

13. In the solution of this question, the whole estate may be represented by a unit, or by any fraction whose numerator is equal to its

and $\frac{3}{8}\frac{4}{3}$ of the residue to another, and the surplus to his relict for her life; the sons' legacies were found to be £257³/₄ different; of how much money did he leave the widow the use?

14. The cargo of a ship being valued at £12 000, and $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of the ship being equal in value to $\frac{1}{9}$ of $\frac{6}{7}$ of $\frac{11}{13}$ of the cargo; what did both ship and cargo stand the owners in?

15. If 248 men, in $5\frac{1}{2}$ days, of 11 hours long, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yards in length, $3\frac{2}{3}$ yards in width, and $2\frac{1}{3}$ yards in depth; in how many days, of 9 hours each, will 24 men dig a trench, of 4 degrees of hardness, $337\frac{1}{2}$ yards long, $5\frac{3}{5}$ yards wide, and $3\frac{1}{2}$ yards deep?

16. A boy, having got 400 nuts, in his return home

Occasional Directions for the Learner.

denominator. Then if $\frac{34}{83}$ of the estate be devised to one son, the residue of the estate will be $\frac{83}{83} - \frac{34}{83}$, or $\frac{49}{83}$. Of this residue, the other son was, per question, to have $\frac{34}{83}$, that is, $\frac{34}{83}$ of $\frac{49}{83}$, or $\frac{1666}{6889}$; and the surplus, or widow's share, is, consequently, $\frac{6889}{6889} - \frac{34}{83} + \frac{1666}{6889}$, or $\frac{6889}{6889} - \frac{4488}{6889} = \frac{2401}{6889}$. Also the difference between the sons' legacies is $\frac{34}{83} - \frac{1666}{6889}$, or $\frac{1156}{6889}$, which fraction is, per question, equal in value to £257 $\frac{1}{6}$. Then, as the difference between the sons' legacies, fractionally expressed, is to £257 $\frac{1}{6}$; so is the widow's share, to the money required.

14. Find, by the Rule-of-Three, the value of $\frac{1}{9}$ of $\frac{6}{7}$ of $\frac{11}{13}$ of the cargo, the whole of which, being represented by a unit, is per question equal to £12 000. Then say, as $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of the ship (reducing the given compound fraction to a simple one), is to that value; so is the whole ship (designated by a unit), to the value thereof. Lastly, add the value of the cargo, as given in the question, to the value of the ship, as ascertained by the last stating, and the sum will be the answer to the question.

16. The first step in the solution of this question is as follows:

	Nuts.
From - - - - -	- 400
Subtract $\frac{5}{12}$ of 400, or - - -	- $166\frac{2}{3}$
First remainder,	- <u><u>233$\frac{1}{3}$</u></u>

was met by A., who took from him $\frac{5}{12}$ of his whole stock; B. lights on him afterwards, and forcibly takes from him $\frac{1}{4}$ of the remainder; unluckily C. found him, and required $\frac{119}{200}$ of what he had left; now D. was, by promise, to have $\frac{3}{4}$ of $\frac{1}{4}$ of the nuts he brought home; how many then had the boy left?

DECIMAL FRACTIONS. *

A Decimal Fraction is one whose denominator is either 10, 100, 1000, &c. according as the numerator consists of one, two, three, or any greater number of figures. The denominator, in decimals, is not expressed, but understood, the numerator only being written, having a dot or inverted period, called the decimal point, placed before it; as $\cdot 2 = \frac{2}{10}$; $\cdot 75 = \frac{75}{100}$; and $\cdot 136 = \frac{136}{1000}$, the last of which may be resolved into its several constituent parts, as follows: $\frac{1}{10}$, $\frac{3}{100}$, and $\frac{6}{1000}$, or $\frac{100}{1000}$, $\frac{30}{1000}$, and $\frac{6}{1000}$, whose sum is $\frac{136}{1000}$; or, according to the decimal notation, $\cdot 1$, $\cdot 03$, and $\cdot 006$, whose sum is $\cdot 136$.

* The doctrine of Decimals is considered as a modern improvement in Fractions, though not of a very recent date. The first hint seems to have been given by John Muller, sometimes named Regiomontanus, about the year 1464; and the first treatise written professedly on this subject, was published at Leyden, in the year 1585, by Simon Stevens, entitled DISME, or DECIMALS; which he informs us, in his Geography, he believes to have been in use among the Indians, and other Eastern nations, long before the sexagesimal notation was introduced by Ptolemy in the time of M. Aurelius.

A Decimal Fraction is, in fact, only an artificial method of expressing a particular kind of Vulgar Fractions, without their denominators.

Occasional Directions for the Learner.

The subsequent parts of the solution are exactly similar to the above. — If the number of nuts, mentioned in the question, be changed from 400 to 4000, the last remainder, in that case, will be $575\frac{55}{64}$ nuts.

Ciphers * placed on the left hand of whole numbers, or on the right hand of decimals, are, of themselves, perfectly insignificant, and make no alteration in their respective values ; for $\cdot 7$, $\cdot 70$, $\cdot 700$, or their equivalents, $\frac{7}{10}$, $\frac{70}{100}$, $\frac{700}{1000}$, have no difference in their value. On the contrary, if ciphers are placed on the left hand of decimals, having the decimal point before the ciphers, the value of the significant figure or figures is thereby diminished in a decimal or tenfold proportion ; as $\cdot 5$, $\cdot 05$, and $\cdot 005$, are respectively equal to $\frac{5}{10}$, $\frac{5}{100}$, and $\frac{5}{1000}$ parts of a unit.

A whole number and a decimal may, at any time, be changed into a mixed number, by writing the decimal part without the point, and subscribing the proper denominator, as follows : $175\cdot 83 = 175 \frac{83}{100}$.

When no decimal point is used, the number is always understood to be wholly integral.

In the following rules of Addition, Subtraction, Multiplication, and Division of Decimals, the arithmetical process, or operative part, is, in general, performed the same as in whole numbers ; the chief object in each rule, especially in Multiplication and Division, being the arrangement of the numbers, and the right placing of the decimal point, which separates or divides the Decimals from the Integers.

* This word is sometimes written with a *y* ; but always improperly : since Dr. Johnson invariably spells it with an *i* : and in the various examples which he produces from different authors, the same mode of spelling the word uniformly prevails.

ADDITION

OF

DECIMAL FRACTIONS.

RULE. — Write the given numbers under one another, according to the respective value of their places, having all the decimal points standing underneath each other. Then add them together as in whole numbers, making the decimal point, in the total, directly under the other points.

EXAMPLE FOR ILLUSTRATION.

Required the sum of $36.974 + 29.365 + .067313 + 238 + 1789.333$.

$$\begin{array}{r}
 36.974 \\
 29.365 \\
 .067313 \\
 238. \\
 1789.333 \\
 \hline
 2093.739313 \text{ the sum.} \\
 \hline
 \hline
 \end{array}$$

EXAMPLES FOR PRACTICE.

1. Find the sum of $375.49 + 324 + 61903.8 + 528 + .5041$.
2. What is the sum of $6429 + 21.371 + 4.1053 + 846.07 + .00425$?
3. What is the sum of $.8705 + .125 + .493 + 26025 + .61802 + .1158$?
4. Required the sum of $455 + 5.173 + .0364 + 112.75 + .065 + 98.743$.
5. Find the sum of $39.333 + 4.056 + .98735 + 46.287 + 253.04 + 23.6234$.

SUBTRACTION

OF

DECIMAL FRACTIONS.

RULE. — Write the lesser number under the greater, according to the value of their places. Then subtract as in whole numbers, making the decimal point, in the remainder, directly under the other points.

EXAMPLE FOR ILLUSTRATION.

What is the difference between 81.476 and 9.1067?

$$\begin{array}{r} 81.476 \\ 9.1067 \\ \hline 72.3693 \text{ the difference.} \\ \hline \hline \end{array}$$

EXAMPLES FOR PRACTICE.

1. From 238.73 subtract 24.836.
2. From 7324.736 take 273.36.
3. From 2659.168 take 390.0081.
4. What is the difference between 71.913 and 407?
5. What is the difference between 74.326 and 104.05?

MULTIPLICATION

OF

DECIMAL FRACTIONS.

RULE. — Write the multiplier immediately under the multiplicand, without regarding the situation of the decimal point in either. Then multiply as in whole numbers,

and make as many decimals in the product as there are in both the factors taken together. — If the product does not contain so many figures, prefix ciphers to supply the defect.

EXAMPLES FOR ILLUSTRATION

Multiply 385.1 by 4.67.

$$\begin{array}{r}
 385.1 \\
 4.67 \\
 \hline
 26957 \\
 23106 \\
 15404 \\
 \hline
 1798.417 \text{ the product.}
 \end{array}$$

Multiply .5203 by .00168.

$$\begin{array}{r}
 .5203 \\
 .00168 \\
 \hline
 41624 \\
 83248 \\
 \hline
 .000874104 \text{ the product.}
 \end{array}$$

EXAMPLES FOR PRACTICE.

1. Multiply 68.236 by 3.425.
2. Multiply .53267 by .7153.
3. Multiply .03645 by .0734.
4. Multiply 100.44754 by .00382.
5. To 510.21 add 61.971, and multiply the sum by .05408.
6. Multiply the difference between 56.1 and .217053 by 21.

When the multiplier is 10, 100, 1000, &c. the product will consist of the same figures as the multiplicand, having the decimal point removed as many places towards the right hand as there are ciphers in the given multiplier, annexing ciphers, when necessary.

EXAMPLES FOR ILLUSTRATION.

Multiply $\cdot 01809$ by 100

$$\cdot 01809 \times 100 = 1\cdot 809$$

Multiply $9\cdot 16$ by 10000

$$9\cdot 16 \times 10000 = 91600.$$

EXAMPLES FOR PRACTICE.

7. $2\cdot 173 \times 100 =$

8. $41\cdot 9 \times 10 =$

9. $\cdot 00021 \times 1000 =$

10. $53\cdot 42 \times 10000 =$

Other Examples for the Learner's Exercise.

11. $1 \times \cdot 1 =$

12. $\cdot 1 \times \cdot 1 =$

13. $\cdot 01 \times 1 =$

14. $10 \times \cdot 1 =$

15. $\cdot 1 \times \cdot 01 =$

16. $\cdot 001 \times 100 =$

17. $\cdot 01 \times \cdot 01 =$

18. $\cdot 01 \times 10 =$

19. $\cdot 001 \times \cdot 001 =$

20. $\cdot 01 \times 1000 =$

DIVISION

OF

DECIMAL FRACTIONS.

Make the divisor an integer, by removing the decimal point to the end of it, removing likewise the decimal point in the dividend an equal number of places towards the right hand. Then divide as in whole numbers, and the integers in the dividend will give the integral part of the quotient, and the decimal figures remaining in the dividend, will give the same number of decimals in the quotient.

EXAMPLE FOR ILLUSTRATION.

Divide 345.60414 by 5.36 $536)34560.414(64.478$ the quotient.

2400

2564

4201

4494

206 the remainder.

REM. After the removal of the decimal point, two places, both in the dividend and divisor, the integers in the dividend, viz. 34560, produce 64, in the quotient, which are therefore integers. And as there are three decimal figures remaining in the dividend, the answer contains also three decimals, which may be continued to any proposed number of places, by annexing ciphers to the remainder, and dividing as before.

* * In the following examples, when the quotients are not finite, the number of decimal places to which the answers are required to be found, is given in the margin between parentheses.

EXAMPLES FOR PRACTICE.

1. Divide 67.289544 by 1.764 .
2. Divide 273.50856 by $.39$.
3. Divide 45.3884 by $.0466$.
4. Divide 723.6429 by 5.18 . - - - (4 decimals)
5. Divide $.18$ by 7.133 . - - - (5 dec.)
6. Divide 9.32594 by 1117 . - - - (5 dec.)
7. Divide $.0238$ by 3.62 . - - - (5 dec.)
8. Divide $.0006123$ by 47.1 .
9. Divide $.83$ by $.03$. - - - (2 dec.)
10. Divide 2 by $.008$.
11. Divide 7202.93 by $.14$.
12. Divide 13.72 by 7.7 . - - - (5 dec.)

When the divisor is 10, 100, 1000, &c. the quotient will consist of the same figures as the dividend, having the decimal point removed as many places towards the left hand as there are ciphers in the divisor.

EXAMPLE FOR ILLUSTRATION.

Divide 117.5 by 100.

$$117.5 \div 100 = 1.175.$$

EXAMPLES FOR PRACTICE.

13. $705.25 \div 100 =$

14. $41.9 \div 1000 =$

15. $872 \div 10 =$

16. $.7587 \div 100 =$

When the divisor is an integer, having ciphers at the end, cancel the ciphers, remove the decimal point of the dividend as many places towards the left hand as there are ciphers cancelled, and divide by the remaining figures of the divisor, as before.

EXAMPLE FOR ILLUSTRATION.

Divide 195.3 by 2100.

Or, $1.953 \div 21$

$$\begin{array}{r} 3 \overline{) 1.953} \end{array}$$

$$\begin{array}{r} 7 \overline{) .651} \end{array}$$

.093 the quotient.

EXAMPLES FOR PRACTICE.

17. $41020 \div 32000$

18. $45.5 \div 2170$ - - - (5 dec.)

19. $13.56 \div 800$

20. $61 \div 79000$ - - - (5 dec.)

* * * I have, in the present edition of this Work, entirely omitted the very intricate and prolix rules for Contracted Multiplication and Division of Decimals. The operations are complex, and difficult to be remembered, and, comparatively, of very little real use.

REDUCTION

OF

DECIMAL FRACTIONS.

CASE I.

To reduce any Vulgar Fraction to an equivalent Decimal.

RULE. — Annex ciphers, or suppose them to be annexed, to the numerator of the given fraction, and divide by the denominator till there is no remainder, or till you have found 4 or 5 places of decimals in the quotient, and the result will be the decimal required.

EXAMPLES FOR ILLUSTRATION.

Reduce $\frac{3}{8}$ to an equivalent decimal.

$$\begin{array}{r} 8 \overline{) 3.000} \\ \underline{30} \\ 00 \\ \underline{00} \\ 000 \\ \underline{000} \\ 0 \end{array}$$

By annexing three ciphers, as decimals, to the numerator of the given fraction, we find the quotient, .375, by the method of short division; which quotient is equal to the given vulgar fraction.

Reduce $\frac{579}{2384}$ to a decimal.

2384)579.000, &c. (.24286 +, the answer.

102 20

68 40

20 720

1 6480

2176 the remainder.

REM. When there is a remainder, the quotient, though carried to 5 or 6 places, is not quite equal to the vulgar fraction; but in the application of decimals to real business, 3 or 4 places at most will be sufficiently accurate, as they express the value of the fraction, within one thousandth, or one ten-thousandth part of a unit. — When deci-

mals are to be used as multipliers or divisors, they should be continued a place or two further than when they are merely used in addition or subtraction, because a small error, in those cases, may affect the result very materially

EXAMPLES FOR PRACTICE.

Reduce the following vulgar fractions to their equivalent decimals.

$$\begin{array}{lcl} 1. & \frac{1}{4} & = \\ 2. & \frac{1}{2} & = \\ 3. & \frac{3}{4} & = \\ 4. & \frac{3}{5} & = \\ 5. & \frac{5}{8} & = \\ 6. & \frac{9}{16} & = \end{array}$$

$$\begin{array}{lcl} 7. & \frac{5}{24} & = \\ 8. & \frac{1}{25} & = \\ 9. & \frac{8}{29} & = \\ 10. & \frac{14}{191} & = \\ 11. & \frac{1}{758} & = \\ 12. & \frac{275}{3842} & = \end{array}$$

CASE II.

To reduce any given finite Decimal to an equivalent Vulgar Fraction.

RULE. — Make the figures of the given decimal, omitting the decimal point, the numerator of a vulgar fraction, and 1, with as many ciphers annexed to it as there are places in the given decimal, the denominator; and then reduce the fraction to its lowest terms.

EXAMPLE FOR ILLUSTRATION.

Reduce .375 to an equivalent vulgar fraction.

$$.375 = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}, \text{ the fraction required.}$$

EXAMPLES FOR PRACTICE.

Reduce the following decimals to equivalent vulgar fractions.

13. ·5 =	16. ·625 =
14. ·75 =	17. ·5625 =
15. ·06 =	18. ·004 =

CASE III.

To find the value of a given Decimal, in Terms of a lower Denomination.

RULE. — Multiply the given decimal by the number of parts in the next inferior denomination, and cut off as many figures to the right hand as there are places in the given decimal, and the figures to the left, if any, will be integers of that denomination. Proceed, in the same manner, through all the denominations to the lowest, or till the decimals cut off are all ciphers, and the several integers on the left hand will express the value of the given decimal.

EXAMPLE FOR ILLUSTRATION.

What is the value of ·628125 of a pound sterling?

$$\begin{array}{r}
 \cdot 628125 \\
 \times 20 \\
 \hline
 12 \cdot 562500 \\
 \times 12 \\
 \hline
 6 \cdot 7500 \\
 \times 4 \\
 \hline
 3 \cdot 00
 \end{array}$$

Answer 12s. 6¾d.

The examples in this Case may be proved, by reducing the given decimal, by the preceding Case, to an equivalent vulgar fraction, and then finding, by Case 6, in Reduction of Vulgar Fractions, the value of that fraction.

Proof of the foregoing Example.

First, $\cdot 628125 = \frac{628125}{1000000} = \frac{5025}{8000} = \frac{201}{320}$ of a pound.

Then divide £201 by the component parts of 320.

$$\begin{array}{r} \text{£} \\ 4)201 \\ \hline 8) \ 50 \text{ " } 5 \\ \hline 10) \ 6 \text{ " } 5 \text{ " } 7\frac{1}{2} \\ \hline \text{£}0 \text{ " } 12 \text{ " } 6\frac{3}{4} \text{ the answer as above.} \end{array}$$

EXAMPLES FOR PRACTICE.

19. What is the value of $\cdot 33125$ of a pound?
20. Required the value of $\cdot 575$ of a shilling.
21. What is the value of $\cdot 67125$ of a pound?
22. What is the value of $\cdot 375$ of a guinea?
23. Find the value of £3.734375.
24. What is the value of $\cdot 6875$ of a lb. avoirdupois?
25. What is the value of $\cdot 5625$ cwt?
26. What is the value of $\cdot 4375$ of an acre?
27. What is the value of $\cdot 9526$ of a hhd. of wine?
28. What is the value of $\cdot 8746$ of a barrel of ale or beer?

CASE IV.

To find the value of any given Decimal of a Pound Sterling, in one line, by inspection only.

RULE. — Double the figure in the place of tenths, for shillings, and add one more to the number, if the second figure of the decimal be 5 or more than 5; then reckon the remaining figures in the second and third places of the decimal, as so many farthings, — abating one farthing, when they are 13 or more than 13; and two farthings, when the number is 38 or more.

The following are the principal heads of this rule : —

- 1 = two shillings.
- 05 = one shilling.
- 025 = sixpence.
- 001 = one farthing, nearly.
- 013 = twelve farthings, nearly.
- 038 = thirty-six farthings, nearly.

EXAMPLE FOR ILLUSTRATION.

What is the value of •876 of a pound?

$$£\cdot876 = 17s. 6\frac{1}{4}d.$$

Here, the first figure in the place of tenths, doubled, makes 16 shillings, to which add one more, for 5 in the place of hundredths, and the result is 17 shillings.

The surplus figures, remaining in the second and third places, being 26, we abate 1, and the remainder, 25, is farthings, making, with the first result, 17s. 6 $\frac{1}{4}$ d., the value required.

EXAMPLES FOR PRACTICE.

Find the values of the following decimals of a pound, by inspection only.

£			£		
29.	•375	=	35.	•885	=
30.	•7	=	36.	•093	=
31.	•25	=	37.	•606	=
32.	•75	=	38.	•007	=
33.	•362	=	39.	•2686	=
34.	•473	=	40.	•34726	=

CASE V.

To reduce Decimals or Integers of a lower Denomination to equivalent Decimals of a higher Denomination.

RULE. — Divide the given decimal or integer by the proper divisor or divisors, as in reduction of whole numbers, and the result will be the decimal required.

EXAMPLE FOR ILLUSTRATION.

Reduce 3d. to the decimal of a pound.

$$\begin{array}{r|l} 12 & 3.00d. \\ 20 & .25s. \\ & \underline{\cdot 0125l.} \text{ the decimal required.} \end{array}$$

Proof. $\pounds \cdot 0125 = \pounds \frac{125}{10000} = \frac{10}{800} = \frac{1}{80} = 3d.$

In this example, the last quotient, $\pounds \cdot 0125$, is found by cancelling the cipher at the end of the divisor, and then removing the decimal point of the dividend, 25, one place towards the left hand : thus,

$$\begin{array}{r} 2\phi) \cdot 025 \\ \underline{\cdot 0125} \text{ the quotient, as above.} \end{array}$$

EXAMPLES FOR PRACTICE.

41. Reduce 13 shillings to the decimal of a pound.
42. Reduce 7.25 shillings to the decimal of a pound.
43. Reduce 6.75d. to the decimal of a pound.
44. Reduce .06q. to the decimal of a shilling.
45. Reduce 11 oz. avoirdupois to the decimal of a lb
46. Reduce 14 poles to the decimal of an acre.
47. Reduce 237.6 yards to the decimal of a mile.

Occasional Directions for the Learner.

47. Reduce $\frac{237.6}{1760}$ of a mile to its lowest terms; then divide the numerator by its denominator, and the quotient will be the decimal required.

48. Reduce 66·5625 yards to the denomination of Eng. ells.

49. Reduce 53·25 Eng. ells to the denomination of Fl. ells.

50. Reduce 88·75 Fl. ells to yards.

CASE VI.

To reduce a Number, consisting of different Denominations, to an equivalent Decimal of a higher Denomination.

RULE. — Begin with the lowest denomination, and reduce it to a decimal of the next superior denomination, prefixing the integer of that denomination. Reduce this mixed number, in the same manner, to the next higher denomination, prefixing the integer of that denomination to the result; and so proceed to the highest or required denomination, and the last result will be the decimal required.

EXAMPLE FOR ILLUSTRATION.

Reduce 15s. 9 $\frac{3}{4}$ d. to the decimal of a pound.

$$\begin{array}{r|l}
 4 & 3 \text{ q.} \\
 \hline
 12 & 9 \cdot 75 \text{ d.} \\
 \hline
 20 & 15 \cdot 8125 \text{ s.} \\
 \hline
 & \underline{\underline{\cdot 790625 \text{ l.}}}, \text{ the Answer.}
 \end{array}$$

First, divide 3 q. by 4, supplying the ciphers mentally, to complete the quotient, and prefix 9d. to the result. Then divide the mixed number, 9·75d., by 12, and the quotient is ·8125 of a shilling. Lastly, prefix 15s., divide the whole by 20, and the quotient is £·790525, the answer.

EXAMPLES FOR PRACTICE.

51. Reduce 14s. 9d. to the decimal of a pound.
52. Reduce 7s. 4½d. to the decimal of a pound.
53. Reduce 5¼d. 6q. (= 5d. 1.6q.) to the decimal of a shilling.
54. Reduce £60 11 6 to pounds.
55. Reduce 2 cwt. 3 q. 21 lb. to cwt.
56. Reduce 9 oz. 9⅓ dr. avoirdupois to the decimal of a pound.
57. Reduce 31 gal. 1.9424 qt. of ale to the decimal of a barrel.
58. Reduce 53 Eng. ells, 2 q. 3½ n. to the denomination of yards.

CASE VII.

To reduce Shillings, Pence, and Farthings, by inspection only, in one line, to the Decimal of a Pound.

RULE. — When the given number of shillings is even, write half its number for the first decimal figure, reserving 5 for the second place of decimals, on account of 1 shilling, when the number of shillings is odd. Reduce the pence and farthings to farthings, to be written in the second and third place of decimals, increasing the last decimal by 1, when the number of farthings is 12, or more than 12; and by 2, when the number of farthings is 36, or more.

Occasional Directions for the Learner.

54. Reduce 11s. 6d. to the decimal of a pound, prefixing £60 to the last result, for the answer.

The following are the heads of this rule.

2 s., 4 s., 6 s., &c. = .1, .2, .3, &c. of a pound, respectively.

1 s. = .05 of a pound, and 6 d. = .025 of a pound.

3 d. = .0125 of a pound, and 9 d. = .0375 of a pound.

1 q. = .001 l., nearly.

3 d. = .013 l., nearly.

9 d. = .038 l. nearly.

EXAMPLES FOR PRACTICE.

Reduce the following sums to decimals of a pound, in one line, by inspection.

59.	16 s. =		63.	6 s. 2 d. =
60.	15 s. =		64.	1 s. 9 $\frac{1}{4}$ d. =
61.	17 s. 4 $\frac{1}{2}$ d. =		65.	19 s. 0 $\frac{3}{4}$ d. =
62.	13 s. 8 $\frac{3}{4}$ d. =		66.	10 $\frac{1}{4}$ d. =

THE RULE-OF-THREE

IN

DECIMALS.

RULE. — State the question as in whole numbers, and reduce the first and third term to the same denomination, and the second term (if a compound number) to one denomination. Then divide the product of the second and third term by the first term, and the quotient will be the answer, in the same denomination as the second term.

REM. If the answer be in pounds sterling, continue the quotient to 5 places of decimals; if in shillings, to four places; if in pence, to 3; and if in farthings, to 2. In other denominations, the number of decimal figures is generally left to the discretion of the Learner, observing, that in the higher names more decimal places are requisite, in order to obtain the answer accurately in the inferior denominations.

QUESTION FOR ILLUSTRATION.

If $1\frac{1}{4}$ yard of cloth cost 15s. $4\frac{1}{2}d.$ what cost 15 pieces, each 21·5 yards?

First, $1\frac{1}{4}$ yd. = 1·25 yd. the first term, and $21\cdot5$ yd. $\times$ 15 = 322·5 yd. the third term; also, 15s. $4\frac{1}{2}d.$ = 15·375s. the second or middle term.

Yd.	s.	Yd.
Then, as 1·25	15·375	322·5
5·	61·5	12·3
1· (By abbrev.)	12·3	9675
		38700
		20)3966·75
		198l. 6·75s.
		12
		9·00d.

Answer, £198 " 6 " 9.

Another way.

Yd.	s.	£	Yd.
As 1·25	15·375	(= ·76875)	322·5
$\frac{\cdot76875 \times 322\cdot5}{1\cdot25} = \text{£}198\cdot3375 = \text{£}198 \text{ " } 6 \text{ " } 9, \text{ the answer.}$			

QUESTIONS FOR EXERCISE.

1. Suppose A. owes B. £296 " 17 " 0, and that he compounds with him for 7s. 6d. in the pound; what will B. receive for his debt?

2. What is the value of $12\frac{1}{2}$ hhd. of wine, at 6s. 3d. per gallon?

3. If $1\frac{1}{2}$ oz. of silver cost $7\frac{4}{5}s.$; how many pounds may be bought for £30 " 5 " 3?

Occasional Directions for the Learner.

3. Find the answer in lb. and decimals of a lb., continuing the latter to 5 places.

4. If $1\frac{2}{5}$ lb. of mace be worth 14 s. 6 d.; what is the price of 75.31 lb.?

5. If 1.47 cwt. of sugar cost £4.10.0; what is 1.7 lb. worth at the same rate?

6. Bought 3 cwt. 1.5 qr. of cloves, at 2 s. 9 d. per lb., and sold the whole for £60.11.6; what did I gain or lose by the bargain?

7. What is the value of 3 hhd. of tobacco, each 4 cwt. 2 qr. 7.4 lb. if 4.2 lb. cost 8 s. 2.3 d.?

8. A merchant bought 436 yards of cloth, at 8 s. 6 d. per yard, and sold it again for 10.75 s. per yard; what was the whole gain?

9. Bought 3 hhd. of tobacco, each weighing 4 cwt. 1.9 qr. at £5.12.0 per cwt., which I sold again at £7.8 per cwt.; what did I gain by the whole?

10. If 1 yd. 2 qr. of velvet cost 15 s. 3 d.? what must be given for 33 yd. 2 qr. 3 n.?

11. What is the commission of £475.17.0, at £3.5 per cent.?

12. What is the insurance of £892.15.6, at 7 guineas per cent.?

13. A person bought a piece of cloth for £6.13.12 s., I desire to know how many yards there were in the same, when he gave after the rate of 4 s. 3 d. per yard?

14. If 1.6 cwt. of sugar cost £3.12.76 s.; what will 3 hhd. cost, each 11 cwt. 3 qr. 10.12 lb.?

15. C. delivered 25.6 ells of holland, at 4 s. 6 d. per ell,

Occasional Directions for the Learner.

5. Find the answer in pence, and 3 decimals of a penny.

7. Reduce the second term to pence, and the answer to shillings and pounds.

10. Reduce the second term to shillings, and the answer to pounds.

13. Find the answer in yards, and 3 decimals of a yard.

15. As 40.7 yd. is to 4.5 s. $\times$ 25.6; so is 1 yd. to the answer in shillings.

to D., for 40·7 yd. of cloth; what was the cloth per yard?

16. A dealer in oil bought 1 T. 3 hhd. 12 gal. of oil for £96^{..}5^{..}3, of which 24·5 gallons by misfortune leaked out; how must he sell the remainder per gallon, to be no loser?

PRACTICE.

CASE I.

When the given Price is Two Shillings.

RULE. — Remove the decimal point one place towards the left hand, for the answer in pounds.

EXAMPLE FOR ILLUSTRATION.

What is the amount of $247\frac{1}{4}$ lb. at 2s. per lb.?

First, $247\frac{1}{4}$ lb. = 247·25 lb.

Then, £24·725 = £24^{..}14^{..}6, the answer.

EXAMPLES FOR PRACTICE.

- | | | |
|----------------------------|--|----------------------------|
| 1. 128 at 2s. | | 3. $656\frac{3}{4}$ at 2s. |
| 2. $478\frac{1}{2}$ at 2s. | | 4. $912\frac{7}{8}$ at 2s. |

CASE II.

When the given Price is an aliquot Part, or can be divided into aliquot Parts, of Two Shillings.

RULE. — Remove the decimal point, as in the first Case, and divide by the aliquot part, or the aliquot parts of which the given price is composed.

EXAMPLES FOR ILLUSTRATION.

What is the amount of $298\frac{1}{2}$ yd., at 2d. per yard?

$$298\frac{1}{2} \text{ yd.} = 298.5 \text{ yd.}$$

$$2d. \left| \frac{1}{12} \right| \begin{array}{r} 29.85 \\ \hline 2.4875 l. = £2 \text{ " } 9 \text{ " } 9, \text{ the answer.} \end{array}$$

What is the amount of 966 yards, at $3\frac{1}{2}$ d. per yard?

$$\begin{array}{l} 3d. \\ \frac{1}{2}d. \end{array} \left| \frac{\frac{1}{8}}{\frac{1}{6}} \right| \begin{array}{r} 96.6 \\ \hline 12.075 \\ 2.0125 \\ \hline 14.0875 l. = £14 \text{ " } 1 \text{ " } 9, \text{ the answer.} \end{array}$$

EXAMPLES FOR PRACTICE.

- | | |
|---|--|
| 5. 968 at 3d. | 15. 248 at 2s. $3\frac{1}{2}$ d. |
| 6. $376\frac{1}{2}$ at 4d. | 16. $361\frac{1}{8}$ at 2s. 4d. |
| 7. $657\frac{1}{8}$ at 6d. | 17. 473 at 6s. |
| 8. $295\frac{1}{2}$ at 8d. | 18. $677\frac{1}{2}$ at 8s. |
| 9. 564 at $4\frac{1}{2}$ d. | 19. 647 at 12s. 8d. |
| 10. 570 at 5d. | 20. $963\frac{1}{4}$ at 14s. 3d. |
| 11. 346 at 9d. | 21. $875\frac{1}{2}$ at 18s. $2\frac{1}{2}$ d. |
| 12. $288\frac{1}{2}$ at $9\frac{3}{4}$ d. | 22. 40 yd. 1 q. 2 nl. at |
| 13. 474 at 11d. | £1 " 16 " 0. |
| 14. $640\frac{1}{8}$ at 18d. | |

CASE III.

When the given Price is any aliquot Part, or can be divided into two or more aliquot Parts of a Pound.

RULE. — Divide the given quantity by such part, or parts, and the result will be the answer in pounds.

EXAMPLE FOR ILLUSTRATION.

What is the value of 2771 yards of cloth, at 3*s.* 4*d.* per yard?

$$3s. \ 4d. \ \left| \frac{1}{6} \right| \begin{array}{r} 2771 \cdot \\ \hline 461 \cdot 833, \text{ \&c.} \\ \hline \end{array} = \text{£}461 \text{ „ } 16 \text{ „ } 8, \text{ the answer.}$$

EXAMPLES FOR PRACTICE.

- | | |
|------------------------------------|-------------------------|
| 23. 481½ at 16 <i>d.</i> | 27. 14 cwt. 1 q. 7 lb. |
| 24. 953 at 2 <i>s.</i> 6 <i>d.</i> | at £1 „ 17 „ 2. |
| 25. 642 at 7 <i>s.</i> 6 <i>d.</i> | 28. 16 cwt. 1 q. 14 lb. |
| 26. 51 cwt. 3 q. at | at £6 „ 13 „ 0. |
| £2 „ 13 „ 4. | |

CIRCULATING DECIMALS.

A circulate, or repeating decimal, is that in which one or more figures continually recur.

A single circulate is that in which a single digit only repeats ; as .666, &c.

A compound circulate is that in which two or more figures repeat ; as .642 642, &c.

A mixed circulate is that which consists of a terminate part and a repetend ; as 86.325 325, &c. and 4.333, &c.

A single or compound circulate may be reduced to an equivalent vulgar fraction, by making the repetend the numerator, removing the decimal point as many places towards the right hand as there are figures in the repetend ; the denominator also to consist of the same number of nines, as .333, &c. = $\frac{3}{9}$ or $\frac{1}{3}$; .999, &c. = $\frac{9}{9} = 1$; and .123 123, &c. = $\frac{123}{999}$ or $\frac{41}{333}$.

$$\text{Also, } .03333 \text{ \&c.} = \frac{.3}{9} = \frac{.1}{3} = \frac{1}{30}.$$

$$.064064 \text{ \&c.} = \frac{64}{999}.$$

$$.006006 \text{ \&c.} = \frac{.06}{9} = \frac{.02}{3} = \frac{2}{300} = \frac{1}{150}.$$

$$.00706706 \text{ \&c.} = \frac{7.06}{999} = \frac{706}{99900} = \frac{353}{49950}.$$

A mixed circulate may be reduced to an equivalent vulgar fraction, by finding, as before, a vulgar fraction equal to the circulating part, and annexing the result to the finite part; as $4.233 \text{ \&c.} = 4\frac{2}{9}$ or $4\frac{1}{3}$; $2.545454 \text{ \&c.} = 2\frac{54}{99}$ or $2\frac{6}{11}$; and $.5333 \text{ \&c.} = .5\frac{3}{9} = .5\frac{1}{3} = \frac{1.6}{3} = \frac{16}{30} = \frac{8}{15}$.

$$\text{Also, } 5.2434343 \text{ \&c.} = 5.2 + .0434343 \text{ \&c.} = 5.2\frac{43}{99} = \frac{519.1}{99} = \frac{5191}{990}.$$

* * Having premised these brief observations on the nature of Circulating Decimals, in general, I shall refer those who may wish to see the investigation of all the rules and peculiar processes used in this part of Decimal Arithmetic, to a Treatise, published in the year 1777, by the late Dr. Henry Clarke, entitled 'The Rationale of Circulating Numbers,' which contains a complete analysis of the whole doctrine of Circulates, both in theory and practice.

DUODECIMALS.

Feet, inches, seconds, &c. are called Duodecimals, because they decrease by twelves, from the place of feet, towards the right hand, as in the following table.

12 Inches	-	-	-	-	make 1 Foot,
12 Seconds	-	-	-	-	— 1 Inch,
12 Thirds	-	-	-	-	— 1 Second,
12 Fourths	-	-	-	-	— 1 Third.

Inches or primes, seconds, &c. are marked thus:— inches or primes ('), seconds ("), thirds (""), fourths ("""), &c.

Multiplication of Duodecimals, or, as it is generally called, Cross Multiplication, is used by artificers and workmen, in casting up the contents of their work.

Dimensions are generally taken in feet and inches, and sometimes in feet, inches, and parts.

RULE. — 1. Arrange the terms of the multiplier under the corresponding names of the multiplicand, and multiply each term of the multiplicand, beginning at the lowest, by the feet in the multiplier, setting down the result of each product under its respective term, carrying one for every twelve, from each lower denomination to the product of the next higher denomination.

2. Multiply each term of the multiplicand, as before, by the inches in the multiplier, and write the result of each product, one place towards the right hand of those names in the multiplicand.

3. Proceed, in the same manner, with the seconds in the multiplier, writing the result of each product, two places removed to the right hand, and for thirds, three places, &c.

Lastly, add all the products together, and the sum will be the answer required.

EXAMPLE FOR ILLUSTRATION.

Multiply 9 f. 3' 4" by 8 f. 9' 7".

$$\begin{array}{r}
 \begin{array}{c} f \quad ' \quad '' \\ 9 \quad '' \quad 3 \quad '' \quad 4 \\ 8 \quad '' \quad 9 \quad '' \quad 7 \\ \hline 74 \quad '' \quad 2 \quad '' \quad 8 \\ 6 \quad '' \quad 11 \quad '' \quad 6 \quad '' \quad 0 \\ 5 \quad '' \quad 4 \quad '' \quad 11 \quad '' \quad 4 \\ \hline 81 \quad '' \quad 7 \quad '' \quad 6 \quad '' \quad 11 \quad '' \quad 4 \\ \hline \hline \text{Answer, } 81 \text{ f. } 7' \quad 6'' \quad 11''' \quad 4''''
 \end{array}
 \end{array}$$

When the number of feet in the multiplicand consists of two or more digits.

RULE. — First, multiply by the feet in the multiplier, as before; then proceed as in the Rule of Practice, taking aliquot parts of a foot, &c., out of the multiplicand, for the inches and parts in the multiplier, and the sum of those results will be the answer required.

Cross Multiplication.

EXAMPLE FOR ILLUSTRATION.

Multiply 257 f. 10' 5" by 7 f. 6' 9"

f.	'	"	
257	10	5	
7	6	9	
6'	$\frac{1}{2}$	$\frac{1}{8}$	1805 " 0 " 11
9"	$\frac{1}{8}$	$\frac{1}{8}$	128 " 11 " 2 " 6
			16 " 1 " 4 " 9 " 9
			1950 " 1 " 6 " 3 " 9

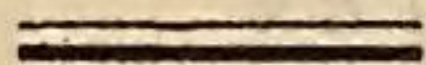
Answer, 1950 f. " 1' " 6" 3''' " 9'''

EXAMPLES FOR PRACTICE.

1. Multiply 9 f. 6' by 4 f. 7'
2. Multiply 12 f. 9' by 6 f. 4'
3. Multiply 5 f. 8' 4" by 8 f. 10'
4. What is the square of 10 f. 5' 6"?
5. Multiply 179 f. 3' by 7 f. 6'
6. Multiply 107 f. 2' 3" by 36 f. 7'
7. Multiply 442 f. 9' 2" 4''' by 2 f. 4' 6"
8. Multiply 10 f. 8' 7" 5''' by 9 f. 4' 9"
9. What is the price of a marble slab, whose length is 5 f. 7' and breadth, 1 f. 10' at 7s. 6d. per foot?
10. What will the glazing of a sash frame come to, at 1s. 6d. per foot, which contains 12 squares, each measuring 1 f. 1' in length, and 11' 6" in breadth?
11. What will the paving of a court-yard come to, at $4\frac{3}{4}$ d. per square yard, the length being 58 f. 6', and the breadth, 54 f.?

INVOLUTION,

OR

THE RAISING OF POWERS.

The power of any number is the product of that number multiplied by itself a certain number of times.

Involution is the method of raising numbers to higher powers.

The number denoting the power is called the index of that power. Thus the index of the square or second power is 2; of the cube, or third power, 3; of the biquadrate or fourth power, 4; and so on.

The product of any number by itself is the square of that number.

If the square be multiplied by the original number, the product will be the cube or third power of that number.

The square of any number multiplied by itself, or the third power multiplied by the original number, will give the fourth power.

The fourth power multiplied by the second, or the third power multiplied by itself, produces the sixth power.

And, in like manner, any higher power may be found; observing, that the index or exponent of the power produced is always equal to the sum of the indices of those powers or factors which are multiplied together.

EXAMPLE FOR ILLUSTRATION.

Required the second, third, and fifth powers of $24\cdot3$.

$$\begin{array}{r}
 24\cdot3 \\
 24\cdot3 \\
 \hline
 729 \\
 5832 \\
 \hline
 59049, \text{ the square of } 24\cdot3. \\
 24\cdot3 \\
 \hline
 177147 \\
 1417176 \\
 \hline
 14348907, \text{ the cube, or third power.} \\
 59049, \text{ the second power.} \\
 \hline
 129140163 \\
 57395628 \\
 129140163 \\
 71744535 \\
 \hline
 847288609443, \text{ the fifth power of } 24\cdot3.
 \end{array}$$

EXAMPLES FOR PRACTICE.

1. What is the second power or square of 416?
2. What is the cube of $7\cdot8$?
3. What is the fourth power of $\cdot045$?
4. What is the fifth power of $\cdot205$?
5. What is the sixth power of $1\cdot49$.
6. What are the respective squares of 4, 8, and 16?
Also of 10, and 20?
7. Required the cubes of 3, 6, 9, and 18. Also of 10,
and 20.
8. What is the tenth power of 10?
9. What is the seventh power of $\frac{3}{4}$?
10. What is the square of $\frac{1\frac{2}{3}}{2\frac{5}{9}}$?
11. What is the eighth power of $\frac{1}{2}$?
12. What is the third power of $7\frac{2}{5}$, or $7\cdot4$?

EVOLUTION,
OR
THE EXTRACTION OF ROOTS.

The root of any given power is such a number as being multiplied into itself a certain number of times, according to the exponent or index of the power, will produce that number.

Evolution, being the reverse of involution, is the method of finding the root of any given power.

SQUARE ROOT.

RULE. — 1. Divide the given number into periods of two figures each, by drawing a small line over them alternately, both ways from the decimal point.

2. Find the greatest square contained in the first two periods, by the subjoined table of square numbers, placing its root on the right hand, and subtracting the square, thus found, from those two periods.

3. To the remainder annex the succeeding period for a new dividend, and place the double of the root for a divisor.

4. Examine how many times the divisor is contained in the dividend, (exclusive of the last figure) and write the result at the next figure of the root, and annex it to the divisor.

5. Multiply the whole divisor by the last found figure of the root, subtract the product, and bring down the next period for a new dividend.

6. To the last divisor add its units' figure for a new divisor, from which find, as before, another figure of the root, and continue the operation, in the same manner, through each succeeding period to the last.

REM. The number of integers in the root is always equal to the number of integral periods in the given square.

If the given number be an imperfect power, or one whose root cannot be accurately found, it is called a surd or irrational quantity ; in which case, the root may be continued to any proposed number of decimals, by bringing down periods of ciphers.

A TABLE OF SQUARE NUMBERS,
From 1 to 9801.

<i>R.</i>	<i>Sqrs.</i>	<i>R.</i>	<i>Sqrs.</i>	<i>R.</i>	<i>Sqrs.</i>	<i>R.</i>	<i>Sqrs.</i>
1	1	26	676	51	2601	76	5776
2	4	27	729	52	2704	77	5929
3	9	28	784	53	2809	78	6084
4	16	29	841	54	2916	79	6241
5	25	30	900	55	3025	80	6400
6	36	31	961	56	3136	81	6561
7	49	32	1024	57	3249	82	6724
8	64	33	1089	58	3364	83	6889
9	81	34	1156	59	3481	84	7056
10	100	35	1225	60	3600	85	7225
11	121	36	1296	61	3721	86	7396
12	144	37	1369	62	3844	87	7569
13	169	38	1444	63	3969	88	7744
14	196	39	1521	64	4096	89	7921
15	225	40	1600	65	4225	90	8100
16	256	41	1681	66	4356	91	8281
17	289	42	1764	67	4489	92	8464
18	324	43	1849	68	4624	93	8649
19	361	44	1936	69	4761	94	8836
20	400	45	2025	70	4900	95	9025
21	441	46	2116	71	5041	96	9216
22	484	47	2209	72	5184	97	9409
23	529	48	2304	73	5329	98	9604
24	576	49	2401	74	5476	99	9801
25	625	50	2500	75	5625	—	—

EXAMPLE FOR ILLUSTRATION.

What is the square root of 29506624?

$$\begin{array}{r}
 \overline{29} \ \overline{50} \ \overline{66} \ \overline{24} \text{ (5432, the root.} \\
 \underline{29 \ 16} \\
 1083) \ \overline{34} \ \overline{66} \\
 \underline{3 \ 32 \ 49} \\
 10862) \ \overline{2 \ 17} \ \overline{24} \\
 \underline{2 \ 17 \ 24}
 \end{array}$$

EXAMPLES FOR PRACTICE.

1. What is the square root of 155236?
2. What is the square root of 2470.09?
3. Find the square root of 12345.4321.
4. Required the square root of 317.695. [*To seven places of decimals.*]
5. Extract the square root of 9606. [*To six places of decimals.*]
6. Required the square root of 36.00000625. [*To ten places of decimals.*]
7. What is the square root of 2? [*To nine places of decimals.*]
8. What is the square root of 3? [*To nine decimals.*]
9. What is the difference between the square of 2401, and the square root of 2401?
10. What is the square root of 5? [*To sixteen decimals.*]

To find the Square Root of Vulgar Fractions.

RULE. — Reduce the given fraction to its lowest terms; and if it be a mixed number, to an improper fraction. Then extract the square root of the numerator and denominator respectively, for the terms of the root required.

EXAMPLE FOR ILLUSTRATION.

What is the square root of $\frac{75}{147}$?

First, $\frac{75}{147} = \frac{25}{49}$.

Then, $\frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}$, the root.

EXAMPLES FOR PRACTICE.

11. What is the square root of $\frac{48}{300}$?

12. Required the square root of $1606\frac{675}{1369}$

To find the Square Root of Surd Quantities.

RULE. — Reduce the given fraction to a decimal, and find the square root, as before, to the required number of decimals.

EXAMPLE FOR ILLUSTRATION.

What is the square root (to 6 places of decimals) of $\frac{3}{4}$?

First, $\frac{3}{4} = .75$

$\cdot 75 \overline{00} \overline{00}$ &c. ($\cdot 866025 +$, the root.

73 96

1726) 1 04 00

6 1 03 56

173202) 44 00 00

2 34 64 04

1732045) 9 35 96 00

8 66 02 25

69 93 75, the remainder.

EXAMPLE FOR THE LEARNER.

13. What is the square root of $\frac{5}{12}$ of $6\frac{2}{5}$? [*To four places of decimals.*]

MISCELLANEOUS QUESTIONS

IN

SQUARE ROOT.

-
1. What is the mean proportional between 16 and 36 ?
 2. If the area of a circle be 4276·5 square poles ; what is the side of a square [*to four decimals*] equal in area to the given circle.
 3. The area of an irregular fish-pond is 2 ac. 1 r. 15 pls. ; required the side of a square [*to three decimals*] that shall be equal in area to the given fish-pond.
 4. In the midst of a pasture well stored with grass,
I took just one acre to tether my ass ;
How long must the cord be, that, eating all round,
He may not graze more than an acre of ground ?
 5. A circular fish-pond is to be dug in a garden that shall take up just half an acre ; what must the length of the cord be [*in yards, to four decimals*] that strikes the circle ?
-

Occasional Directions for the Learner.

1. Multiply the two given numbers together, and the square root of the product will be the answer.
2. The answer to this question is the square root of 4276·5.
3. Reduce 2 ac. 1 r. 15 pls. into square yards, and the square root of that number will be the answer.
4. Multiply 1·273239 by the given area (4840 *square yards*), and the square root of the product (*to 4 decimals*) will be the length of the diameter. Divide the diameter by 2, and the quotient will be the radius, or length of the cord. — Remark. If the area of a circle be 1, the square of the diameter is 1·273239.

6. Two ships set sail from the same port, the one sails due north 84 leagues, and the other, due west 50 leagues ; required the distance between the two ships. [*To two decimals.*]

7. If a line, 27 yards long, will exactly reach from the top of a fort to the opposite bank of the river, whose breadth is 23 yards ; what is the height of the fort ? [*To three decimals.*]

8. A ladder, 40 feet long, may be so placed that it will reach a window 21 feet high, on one side of a street, and, from the same station, another window 33 feet high, on the opposite side ; required the breadth of the street. [*To three decimals.*]

9. The height of a tree, growing in the middle of a circular island, 30 feet in diameter, is 53 feet ; and a line 112 feet long, reaches from the top of the tree to the opposite side of the water : what is the breadth of the moat ? [*To two decimals.*]

10. At Matlock, in Derbyshire, there is a perpendicular rock, by the side of the river Derwent, whose height I endeavoured to ascertain, and found that the distance between the place of observation and the foot of the rock, was 55.5 yards, and from the said place to the top of the rock, 140.5 yards : what is the height of the rock ? [*To two decimals.*]

11. A may-pole being broke by a blast of wind, the end of the broken part (whose length was 63 feet) struck

Occasional Directions for the Learner.

6. Here the base and perpendicular of a right-angled triangle are given, to find the hypotenuse. The rule is ; Add the squares of the two given sides together, and the square root of the sum will be the length of the hypotenuse.

7. Here the hypotenuse and base are the two given sides, and the rule, in this case, is ; From the square of the hypotenuse, subtract the square of the base, and the square root of the remainder will be the length of the side required.

the ground at the distance of 30 feet from the foot of the may-pole ; what was the height of the whole may-pole ?
[*To four decimals.*]

12. There are two columns in the ruins of Persepolis, standing upright, the one is 64 feet high, and the other 50 feet ; in a straight line between the two columns, stands a statue, whose head is 97 feet from the summit of the higher, and 86 feet from the top of the lower column ; the horizontal distance between the statue and the lower column is 76 feet. Required the height of the statue and its distance from the higher column.
[*To three decimals.*]

CUBE ROOT.

RULE. — 1. Divide the given number into periods of three figures each, both ways, from the decimal point. — The first period, on the left hand, will sometimes consist of only one, or two figures, according to the number of integral digits in the given power.

2. Find the greatest cube number which is contained in the first two periods, from the subjoined table of cubes, and put the root as a quotient, in the answer's place, subtracting the cube thereof from those two periods, and to the remainder annex the subsequent period of the given number, for a resolvend.

3. Let the figures of the root, now ascertained, being the two first figures of the answer, be represented by the letter a ; then $300aa$, that is, the square of a (or the number denoted by that letter) multiplied by 300, will give a divisor. Examine how often that divisor is contained in the resolvend, and the result will be the next figure of the root. Let that new found figure, then, be represented by the letter b . Then will $300aab +$

$30\ abb + bbb$ give the number to be subtracted from the preceding resolvend; to the remainder bring down another period, for a new resolvend, and proceed, as in the first place, to find a new divisor, another figure of the root, and a new subtrahend, — the first three figures of the root being represented by a , and the new or additional figure by b .

REM. 1. The algebraical expression, $300\ aab$, implies that 300 times the square of a , or the number which it denotes, is to be multiplied by the number denoted by b ; and, in like manner, the expression, $30\ abb$, implies, that 30 times a is to be multiplied by the square of b , or when it is more convenient, by its component parts: and bbb , or b^3 , implies that the number denoted by the letter b is to be cubed, or raised to the third power.

2. The number of integers in the root, or answer, is always equal to the number of integral periods contained in the given power which constitutes the question. But in every other part of the solution, the position of the decimal point is totally disregarded, the various operations of multiplication, addition, and subtraction, being managed exactly as in whole numbers.

3. If the subtrahend should, in any case, prove greater than the resolvend, it shows that the last figure of the root was taken too large, and it must of course be diminished.

A TABLE OF CUBE NUMBERS.

*** In the Table of Cubes, published in Donn's Mathematical Essays, to which I referred in a former edition of this Work, there are sixty typographical errata ; on which account, I now present the Learner with a correct Table of Cube Numbers, from 1 to 970 299.*

R.	Cubes.	R.	Cubes.	R.	Cubes.	R.	Cubes.
1	1	26	17 576	51	132 651	76	438 976
2	8	27	19 683	52	140 608	77	456 533
3	27	28	21 952	53	148 877	78	474 552
4	64	29	24 389	54	157 464	79	493 039
5	125	30	27 000	55	166 375	80	512 000
6	216	31	29 791	56	175 616	81	531 441
7	343	32	32 768	57	185 193	82	551 368
8	512	33	35 937	58	195 112	83	571 787
9	729	34	39 304	59	205 379	84	592 704
10	1 000	35	42 875	60	216 000	85	614 125
11	1 331	36	46 656	61	226 981	86	636 056
12	1 728	37	50 653	62	238 328	87	658 503
13	2 197	38	54 872	63	250 047	88	681 472
14	2 744	39	59 319	64	262 144	89	704 969
15	3 375	40	64 000	65	274 625	90	729 000
16	4 096	41	68 921	66	287 496	91	753 571
17	4 913	42	74 088	67	300 763	92	778 688
18	5 832	43	79 507	68	314 432	93	804 357
19	6 859	44	85 184	69	328 509	94	830 584
20	8 000	45	91 125	70	343 000	95	857 375
21	9 261	46	97 336	71	357 911	96	884 736
22	10 648	47	103 823	72	373 248	97	912 673
23	12 167	48	110 592	73	389 017	98	941 192
24	13 824	49	117 649	74	405 224	99	970 299
25	15 625	50	125 000	75	421 875	—	— — —

INTRODUCTORY EXAMPLES.

Suppose $a = 36$; what is the product of $300\ aa$?

$$\begin{array}{r} 36 \\ 36 \\ \hline 216 \\ 108 \\ \hline aa = 1296 \\ 300 \end{array}$$

$300\ aa = \underline{\underline{388800}}$, the first divisor in the solution of the first Example for Illustration.

Again; suppose $a = 36$, and $b = 5$, required the following products, $300\ aab$, $30\ abb$, and bbb .

First, $300\ aa = 388800$

$$b = 5$$

$$300\ aab = \underline{\underline{1944000}}$$

$$a = 36$$

$$30$$

$$30\ a = 1080$$

$$bb = 5 \times 5$$

$$5400$$

$$5$$

$$30\ abb = \underline{\underline{27000}}$$

$$\text{And } bbb = 5 \times 5 \times 5 = 125.$$

N. B. — This last number may always be found from the foregoing Table of cube numbers.

EXAMPLE 1. FOR ILLUSTRATION.

What is the cube root of 48627125?

$$\overline{48}\ \overline{627}\ \overline{125}\ (365$$

$$46\ 656$$

$$388800) \quad 1\ 971\ 125, \text{ resolvend.}$$

$$300\ aab = 1\ 944\ 000$$

$$30\ abb = 27\ 000$$

$$bbb = 125$$

$$\underline{\underline{1\ 971\ 125, \text{ subtrahend.}}}$$

*

Here, $a = 36$
 $b = 5$

Cube Root.

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Ex. 2. What is the cube root of 482 056·280 171 929?

$$\begin{array}{r} \overline{482\ 056\cdot280\ 171\ 929} \text{ (78·409, the root.} \\ \underline{474\ 552} \end{array}$$

(1.) 1825 200) 7 504 280, resolvend.

$$\begin{array}{r} 300\ aab = 7\ 300\ 800 \\ 30\ abb = 37\ 440 \\ bbb = 64 \end{array} \left. \vphantom{\begin{array}{r} 300\ aab = 7\ 300\ 800 \\ 30\ abb = 37\ 440 \\ bbb = 64 \end{array}} \right\} \text{ Here, } a = 78 \\ b = 4$$

7 338 304, subtrahend.

(2.) 18439680000) 165 976 171 929, new resolvend.

$$\begin{array}{r} 300\ aab = 165\ 957\ 120\ 000 \\ 30\ abb = 19\ 051\ 200 \\ bbb = 729 \end{array} \left. \vphantom{\begin{array}{r} 300\ aab = 165\ 957\ 120\ 000 \\ 30\ abb = 19\ 051\ 200 \\ bbb = 729 \end{array}} \right\} \text{ Here, } a = 7840 \\ b = 9$$

165 976 171 929, subtrahend.

*

AUXILIARY CALCULATIONS.

First Part.

$$\begin{array}{r} a = 78 \\ \text{Per Table of squares, } aa = 6084 \\ 300 \\ 300\ aa = \underline{1825200}, \text{ the first divisor.} \\ b = 4 \\ 300\ aa = 1825200 \\ b = 4 \\ 300\ aab = \underline{7300800} \end{array}$$

$$\begin{array}{r} a = 78 \\ 30 \\ 30\ a = 2340 \\ bb = 16 \\ 30\ abb = \underline{37440} \end{array}$$

Per Table of cubes, bbb = 64

Cube Root.

Second Part.

$$a = 784.$$

And here, the divisor, $(300 aa)$ 184396800, being larger than the resolvend 165976171, the quotient figure is 0, and another period, 929, being brought down, a new divisor is found as follows:

$$\begin{array}{r}
 a = 7840 \\
 7840 \\
 7840 \\
 \hline
 54880 \dots \\
 6585600 \\
 \hline
 aa = 61465600 \\
 300 \\
 \hline
 300 aa = \underline{\underline{18439680000}}, \text{ the second divisor.}
 \end{array}$$

$$\begin{array}{r}
 b = 9 \\
 300 aa = 18439680000 \\
 b = 9 \\
 \hline
 300 aab = \underline{\underline{105957120000}}
 \end{array}$$

$$\begin{array}{r}
 a = 7840 \\
 30 \\
 \hline
 30 a = 235200 \\
 bb = 81 \\
 \hline
 235200 \\
 1881600 \\
 \hline
 30 abb = \underline{\underline{19051200}}
 \end{array}$$

Per Table of cubes, $bbb = 729$

EXAMPLES FOR PRACTICE.

1. What is the cube root of 39651821?
2. What is the cube root of 5725732069?
3. What is the cube root of 349359908.507?
4. Required the cube root of 27189441343.
5. What is the cube root of 72147.158747?
6. Extract the cube root of .000002504374712.

7. Extract the cube root of 345815367977·871.
8. Required the cube root, to 3 places of decimals, of 8232·1032709.
9. What is the cube root, to 5 decimals in the root, of 171?
10. What is the difference between the cube, and the cube root, of 1000?
11. The solidity of a certain sphere is 11390·655 cubic feet; required the lineal side of a cube of equal solidity.
12. The lineal side of a cubic piece of marble is 36 feet, required the lineal side of another cubic piece whose solidity is $6\frac{1}{3}$ times as large.

To find the Cube Root of Vulgar Fractions.

RULE. — Reduce the given fraction to its least terms; if it be a mixed number, to an improper fraction; and extract the cube roots of the numerator and denominator respectively, for the terms of the root required.

EXAMPLE FOR ILLUSTRATION.

What is the cube root of $\frac{972800}{37397700}$?

$$\text{First, } \frac{972800}{37397700} = \frac{9728}{373977} = \frac{512}{19683}$$

$$\text{Then, } \frac{\sqrt[3]{512}}{\sqrt[3]{19683}} = \frac{8}{27}, \text{ the answer.}$$

Occasional Directions for the Learner.

11. The cube root of 11390·655 will be the lineal side of the cube required, in feet.
12. The cube of 36 will be the solidity of the first-mentioned piece of marble; which multiply by $6\frac{1}{3}$, and extract the cube root of the product, for the answer in feet.

EXAMPLES FOR PRACTICE.

13. What is the cube root of $\frac{162}{750}$?
 14. What is the cube root of $2\frac{10}{27}$?
 15. What is the cube root of $405\frac{112}{500}$?

Of Surd Quantities.

Reduce the given fraction to a decimal, and extract the cube root, as before, to the required number of decimals.

EXAMPLE FOR ILLUSTRATION.

What is the cube root, to three places of decimals, of $5\frac{3}{5}$?

First, $5\frac{3}{5} = 5\frac{6}{10}$, or 5.6 .

$$\begin{array}{r}
 \overline{5.600\ 000\ 000} (1.775 \\
 \underline{4\ 913} \\
 86700) 687\ 000, \text{ resolvend.} \\
 \underline{606\ 900} \quad a=17 \\
 24\ 990 \quad b=7 \\
 \underline{343} \\
 632\ 233, \text{ subtrahend.} \\
 9398700) 54\ 767\ 000, \text{ resolvend.} \\
 \underline{46\ 993\ 500} \quad a=177 \\
 132\ 750 \quad b=5 \\
 \underline{125} \\
 47\ 126\ 375, \text{ subtrahend.} \\
 \underline{7\ 640\ 625}, \text{ the remainder.}
 \end{array}$$

EXAMPLES FOR PRACTICE.

16. What is the cube root, to 4 places of decimals, of $13\frac{2}{3}$?
 17. What is the cube root, to 3 places of decimals, of $7\frac{5}{7}$?

INTEREST.

Interest is the money paid or allowed by the borrower to the lender, for the use or loan of money, for a certain time, according to a fixed rate per centum per annum.

The principal is the sum of money lent, and on which the interest is reckoned.

The rate is the sum per cent. per annum, agreed upon for the calculation of the interest?

Simple interest is that which arises from the principal only, at a certain rate per cent. per annum, whether it be for one, two, or any greater number of years.

The amount is the sum of the principal and interest added together.

REM. From the year 1714 to the present time, the rate of interest, as established by law, is £5 for £100, for one year. In the colonies belonging to this country, however, and in many other parts of the world, a much higher rate of interest is allowed.

CASE I.

To find the Interest of any given Sum, for a Year, at any proposed Rate per Cent.

RULE 1. — Common way. Multiply the principal by the rate per cent., divide the product by 100, and the quotient will be the interest required.

2. — By Practice. Of the given principal take the aliquot part or parts of 100, for the rate per cent., and the result will be the interest required.

A TABLE
Of the Aliquot Parts of 100.

Rates.	Aliquot Parts of 100.
£5 per cent.	£5 = $\frac{1}{20}$
£4½ per cent.	£5 = $\frac{1}{20}$ £½ = $\frac{1}{10}$ (of £5), <i>subtr.</i>
£4 per cent.	£4 = $\frac{1}{25}$
£3¾ per cent.	£5 = $\frac{1}{20}$ £1¼ = $\frac{1}{4}$ (of £5), <i>subtr.</i>
£3½ per cent.	£2½ = $\frac{1}{40}$ £1 = $\frac{1}{100}$, <i>add.</i>
£3 per cent.	£2 = $\frac{1}{50}$ £1 = $\frac{1}{2}$ (of £2), <i>add.</i>
£2½ per cent.	£2½ = $\frac{1}{40}$
£2¼ per cent.	£2 = $\frac{1}{50}$ £¼ = $\frac{1}{8}$ (of £2), <i>add.</i>
£2 per cent.	£2 = $\frac{1}{50}$
£1¾ per cent.	£2 = $\frac{1}{50}$ £¼ = $\frac{1}{8}$ (of £2), <i>subtr.</i>
£1½ per cent.	£2½ = $\frac{1}{40}$ £1 = $\frac{1}{100}$, <i>subtr.</i>
£1¼ per cent.	£1¼ = $\frac{1}{80}$
£1 per cent.	£1 = $\frac{1}{100}$

Simple Interest.

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QUESTION FOR ILLUSTRATION.

What is the interest of £917 ⁶/₁₀₀ ⁸/₁₀₀ for a year, at £3¹/₂ per cent. per annum?

Common Way.

£	s.	d.
917	6	8
		3
1/2	1/2	2752 0 0
		458 13 4
32		10 13 4
		20
2		13
		12
1		60
		4
2		40, or $\frac{40}{100} = \frac{2}{5}$.

Answer, £32 ²/₁₀₀ ^{1 1/2 2/5}/₁₀₀.

By Practice.

		£	s.	d.
		917	6	8
2 1/2	1/40	22 18 8		
1	1/100	9 3 5 1/2 2/5		
		£32 2 1 1/2 2/5	the answer.	

Another Method.

		£	s.	d.
		917	6	8
10	1/10	91 14 8	the interest at £10 per cent.	
2 1/2	1/4	22 18 8		
1	1/10	9 3 5 1/2 2/5		
		£32 2 1 1/2 2/5	the answer.	

EXAMPLES FOR PRACTICE.

1. What is the interest of £837 ⁹/₁₀₀ ⁷/₁₀₀ for a year, at £5 per cent. per annum?

E 3

2. What is the yearly interest of £831 „ 17 „ 6, at £4 $\frac{1}{2}$ per cent. per annum?
3. At £4 $\frac{1}{4}$ per cent. per annum, what is the amount of £876 „ 12 „ 7, for a year?
4. What is the interest of £1834 „ 7 „ 10 $\frac{1}{2}$ for a year, at £3 $\frac{3}{4}$ per cent. per annum?

CASE II.

To find the Interest of a given Sum, for any Number, of Years.

RULE 1. — Common way. Multiply the principal by the rate per cent., and the product by the number of years; then divide by 100, as before, and the quotient will be the interest required.

2. By Practice. Multiply the rate per cent. by the number of years, and for the product take aliquot parts of 100, out of the given principal, and the result will be the interest required.

EXAMPLE FOR ILLUSTRATION.

What are the interest and amount of £371 „ 16 „ 2 for 7 years, at £5 per cent. per annum?

Common Way.

£	s.	d.
371	„ 16	„ 2
		5
1859	„ 0	„ 10
		7
130	13	„ 5 „ 10
	20	
2	65	
	12	
7	90	
	4	
3	60	
	100	= $\frac{3}{5}q$.

£130 „ 2 „ 7 $\frac{3}{4}$ $\frac{3}{5}$, the interest.

By Practice.

First, $\pounds 5 \times 7 = \pounds 35$ per cent.

			£	s.	d.	
			371	16	2	
25	10	$\frac{1}{4}$	92	19	$0\frac{1}{4}$	
		$\frac{1}{10}$	37	3	$7\frac{1}{4}\frac{3}{5}$	
			130	2	$7\frac{1}{2}\frac{3}{5}$	the interest.
			371	16	2	given principal.
			<u>£501</u>	<u>18</u>	<u>$9\frac{1}{2}\frac{3}{5}$</u>	the amount.

EXAMPLES FOR PRACTICE.

5. What is the interest of $\pounds 1089\text{ „ }16\text{ „ }8$ for 5 years, at $\pounds 4$ per cent. per annum?
6. What is the interest of $\pounds 179\text{ „ }15\text{ „ }0$ for 3 years, at $\pounds 5$ per cent. per annum?
7. What is the interest of $\pounds 120\text{ „ }10\text{ „ }0$ for $2\frac{1}{2}$ years, at $\pounds 4\frac{3}{4}$ per cent. per annum?
8. What is the amount of 50 guineas for $4\frac{3}{4}$ years, at $\pounds 4\frac{1}{2}$ per cent. per annum?

CASE III.

To find the Interest of a given Sum, for any Number of Weeks or Days.

RULE. — Find the interest of the given sum for one year; then, by the Rule of Three Direct, As 52 weeks, or 365 days, are to that interest; so is the proposed number of weeks, or days, to the interest required

Occasional Directions for the Learner.

7 and 8. Multiply the principal by the product of the rate per cent. and time, and divide the product by 100, the quotient will be the interest required. In the 8th example, add the interest to the given principal, and the sum will be the required amount.

QUESTIONS FOR ILLUSTRATION.

What is the interest of £142 " 10 " 0 for 52 days, at £4½ per cent. per annum?

5 ½	1 20 10	<table style="border-collapse: collapse; margin: 0 auto;"> <tr> <td style="text-align: right; padding-right: 5px;">£</td> <td style="text-align: center; padding: 0 5px;">s.</td> <td style="text-align: left; padding-left: 5px;">d.</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">142</td> <td style="padding: 0 5px;">"</td> <td style="border-right: 1px solid black; padding: 0 5px;">10</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"></td> <td style="padding: 0 5px;">"</td> <td style="border-right: 1px solid black; padding: 0 5px;">0</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">7</td> <td style="padding: 0 5px;">"</td> <td style="border-right: 1px solid black; padding: 0 5px;">2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"></td> <td style="padding: 0 5px;">"</td> <td style="border-right: 1px solid black; padding: 0 5px;">6</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"></td> <td style="padding: 0 5px;">"</td> <td style="border-right: 1px solid black; padding: 0 5px;">14</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"></td> <td style="padding: 0 5px;">"</td> <td style="border-right: 1px solid black; padding: 0 5px;">3</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">£6</td> <td style="padding: 0 5px;">"</td> <td style="border-right: 1px solid black; padding: 0 5px;">8</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"></td> <td style="padding: 0 5px;">"</td> <td style="border-right: 1px solid black; padding: 0 5px;">3</td> </tr> </table>	£	s.	d.	142	"	10		"	0	7	"	2		"	6		"	14		"	3	£6	"	8		"	3
£	s.	d.																											
142	"	10																											
	"	0																											
7	"	2																											
	"	6																											
	"	14																											
	"	3																											
£6	"	8																											
	"	3																											

interest for 1 yr., or 365 days.

<i>Days.</i>	£ s. d.	<i>Days.</i>
Then, as 365 days	6 " 8 " 3 10 64 " 2 " 6 5 plus 2	52
	365) 333 " 9 " 0 20 6669 3019 99 12 1188 93 4 372 7, the remainder.	(£0 " 18 " 3¼ ⁷ / ₃₆₅ , the interest for 52 days.

QUESTIONS FOR EXERCISE.

9. What is the interest of £871 for 14 weeks, at £5 per cent. per annum?

10. What is the interest of £212 for 1 year and 17 weeks, at £4 per cent. per annum?

Occasional Directions for the Learner.

10. To the interest of the given sum for one year (found by Practice) add the interest for 17 weeks, (found by the Rule of Three,) and the sum will be the answer.

11. What is the interest of £120 for 248 days, at £4 per cent. per annum?

12. What is the amount of £117 for 107 days, at £4 $\frac{3}{4}$ per cent. per annum?

13. What is the interest of £225.10.0, from November the 25th to December 17th, (both days inclusive,) at £4 $\frac{1}{2}$ per cent. per annum?

CASE IV.

To find the Interest of a given Sum, at £5 per Cent., for any Number of Days.

RULE 1. — Common way. Multiply the principal by the given number of days, and divide the product by 7300, and the quotient will be the interest required.

2. — By decimals. Multiply the principal (in pounds and decimals of a pound) by the given number of days, remove the decimal point in the product two places towards the left hand, and divide the result by 73, the quotient will be the interest required.

REM. 1. This method of computing interest for days is manifestly borrowed from the Rule of Five, where the two first terms are £100 and 365 days; the middle term, £5; and the two third terms, the given principal and number of days.

2. The preceding rule for finding the interest of any sum at 5 per cent., for any number of days, when tables of interest are not at hand, is better adapted for practice than those which are in common use.

Simple Interest.

EXAMPLES FOR ILLUSTRATION.

Required the interest of £ 910 *n* 15 *n* 0 at £ 5 per cent., for 27 days.

Common Way.

£	s.	d.	
910	<i>n</i> 15	<i>n</i> 0	
		9	
8196	<i>n</i> 15	<i>n</i> 0	
		3	

7300)24590 *n* 5 *n* 0 (£ 3 *n* 7 *n* 4 $\frac{1}{4}$ $\frac{287}{365}$, the interest.

2690
20
53805
2705
12
32460
3260
4
13040

$\frac{5740}{7300} = \frac{287}{365} q.$

By Decimals.

£	
910.75	
9	
8196.75	
3	

73)245.90 25(3.3685 = £ 3 *n* 7 *n* 4 $\frac{1}{4}$ +

269
500
622
385
20

REM. $3.3685 \frac{20}{3} = £ 3 *n* 7 *n* 4 $\frac{1}{4}$ $\frac{287}{365}$.$

QUESTIONS FOR PRACTICE.

14. What is the interest of £874 for 68 days, at 5 per cent.?

15. What is the interest of £3204^{..}14^{..}0 for 73 days, at 5 per cent.?

16. What is the interest of the following sums, at 5 per cent.; viz. of £500 for 26 days, of £350 for 82 days, and of £150 for 79 days?

COMMISSION AND BROKERAGE.

Commission is an allowance, according to a certain rate per cent., to an agent or correspondent, for buying or selling goods for his employer.

Brokerage is of the same nature as commission, being an allowance, after a certain rate per cent., to a broker, for buying or selling money in the public funds, or for transacting any kind of mercantile business on account of his employer.

Occasional Directions for the Learner.

15. Divide the given principal by 100; or remove the decimal point two places to the left hand;—and the result, either way, will be the interest required. In this question, the multiplier, or given number of days, being exactly equal to the divisor, both are of course omitted. The same answer may be obtained by finding the interest of the given sum for one year, and dividing the result by 5, because 73 days are $\frac{1}{5}$ of a year.

16. Multiply the several sums by the number of days they are respectively at interest, and divide the amount of the products by 7300, as before, and the quotient will be the amount of the interest required.

To find the Commission or Brokerage of any Sum, when the Rate per Cent. is more than a Pound.

RULE. — Multiply the given sum by the rate per cent., and divide the product by 100, for the commission or brokerage required.

When the Rate per Cent. is less than a Pound.

RULE. — Of the given sum take the proper aliquot part or parts of a pound, for the rate per cent., and divide the result by 100 for the answer. Or, Find the commission at 1 per cent., by dividing the given sum by 100, and of that commission take the proper aliquot part or parts of a pound, for the rate per cent., and the result will be the answer.

EXAMPLES FOR ILLUSTRATION.

1. What is the commission of £500 " 13 " 6, at £3½ per cent.?

£	s.	d.
500	13	6
		3
£	1	2
1	2	2
17 52 " 7 " 3		
20		
10 47		
12		
5 67		
4		
2 68		

Answer, £17 " 10 " 5½ ¹⁷/₂₅.

By Practice.

£	s.	d.
500	13	6
£	2	½
1	1	100
12 " 10 " 4 - ¹ / ₅		
5 " 0 " 1½ ¹² / ₂₅		
17 " 10 " 5½ ¹⁷ / ₂₅ , the commission.		

2. Required the brokerage of £420 " 12 " 6, at 6s. 8d. per cent.

First Method.

			£	s.	d.
			420	"	12 " 6
6s. 8d.	$\frac{1}{3}$	1	40	"	4 " 2
			20		
		8	04		
			12		
			50		
			4		
		2	00		

Answer, £1 " 8 " 0 $\frac{1}{2}$.

Second Method.

			£	s.	d.
			420	"	12 " 6
£1.	$\frac{1}{100}$		4	"	4 " 1 $\frac{1}{2}$
6s. 8d.	$\frac{1}{3}$		1	"	8 " 0 $\frac{1}{2}$, the answer.

EXAMPLES FOR PRACTICE.

1. What is the commission of £742 " 15 " 6, at £3 $\frac{1}{2}$ per cent.?
2. What must I allow my correspondent for disbursing on my account £529 " 18 " 5, at £2 $\frac{1}{2}$ per cent.?
3. If I allow my factor £3 $\frac{7}{8}$ per cent. for commission, what may he demand on the laying out of £1200?
4. My correspondent writes me word, that he has bought goods, on my acct., to the value of £474 " 14 " 6; what does his commission come to, at £7 $\frac{5}{8}$ per cent.?
5. What is the brokerage of £915 at 5s. per cent.?
6. Required the brokerage of £3299 " 12 " 6, at £ $\frac{1}{8}$, or 2s. 6d. per cent.?
7. If a broker sell goods to the value of £1017 " 15 " 8; what will his brokerage come to, at £1 $\frac{1}{2}$ per cent.?
8. If I allow my broker £3 $\frac{3}{4}$ per cent., what may he demand, when he has sold goods to the amount of £657 " 4 " 4 $\frac{1}{2}$?

INSURANCE.

Insurance is an allowance or premium, after a certain rate per cent., which is paid to the insurers or underwriters; for which consideration or allowance they engage to answer for the damage or loss of houses, ships, goods, merchandise, &c. by accidents, such as fire, the dangers of the sea, or any other specified cause.

To find the Insurance of a given Sum, at any Rate per Cent.

RULE. — Multiply the value of the property insured by the rate, and divide the product by 100.

EXAMPLE FOR ILLUSTRATION.

What is the insurance of £ 648 " 17 " 2, at £ 2½ per cent.?

		£ s. d.
		648 " 17 " 2
£		648 " 17 " 2
½	½	324 " 8 " 7
		16 22 " 2 " 11
		20
		4 42
		12
		5 15

Answer, £16 " 4 " 5 $\frac{15}{100}$, or $\frac{5}{20}$.

Or thus, by Practice.

		£ s. d.
		648 " 17 " 2
£		648 " 17 " 2
2½	¼	16 , 4 " 5 $\frac{15}{20}$, the insurance.

QUESTIONS FOR PRACTICE.

9. What is the insurance of £2571, at $13\frac{1}{2}$ per cent.?

10. What is the insurance of £1049⁶₂, at $7\frac{3}{4}$ per cent.?

11. What is the insurance of a ship and cargo, valued at £25,616, at $10\frac{5}{8}$ per cent.?

12. What is the insurance of £3575¹⁵₆, for a year, at 3s. 9d. per cent. per annum?

13. Required the insurance of £763¹²₀ at $15\frac{1}{4}$ per cent.?

14. What is the insurance of £700, at 4 guineas per cent.? and how much will the duty amount to, at 2s. 6d. per cent.?

In time of war, insurances are usually done at a much higher price, on condition of returning to the party insuring a certain part of the premium, in case the ship arrives safe at her destined port.

15. What must be paid for the insurance of a ship and cargo, valued at £2800, at 12 guineas per cent., on condition of 5 guineas per cent. being returned if the ship arrive safe? How much will the duty come to, at 5s. per cent., and how much of the premium must be returned by the underwriters, in case the ship arrives at her port without any accident?

Occasional Directions for the Learner.

14. Multiply each of the given rates by the number of hundreds insured, and the products will be the insurance and duty required.

PURCHASING OF STOCKS.

The word Stock originally meant a particular sum of money contributed to the establishing a fund, to enable a company to carry on a certain trade; by the payment of which the person became a partner, and received a share of the profit, in proportion to the money advanced. But this term has been extended further, and signifies, at present, any sum of money lent to government, and constituting a part of the national debt, on condition of receiving a certain rate of interest for the same, according to the nature of the stock, half-yearly, till the money is either repaid by government, or sold to any one who chooses to become a purchaser.

The prices of stocks, or rates per cent., are the several sums for which £100 of those respective stocks sell, at any given time.

REM. The particular stock called '3 per cent. reduced,' as the name imports, yields £3 per cent. per annum on each £100 of stock; which interest is payable, at the Bank of England, in half-yearly dividends, viz. one half at Old Lady-day, April 5, and the other half at Old Michaelmas-day, October 10, in every year.

The '3 per cent. consols,' also, yields 3 per cent. per annum, on every £100 of stock; and this interest, too, is payable at the Bank of England, in half-yearly dividends, viz. at Old Christmas-day, January 5, and at Old Midsummer-day, July 5.

The '4 per cents.,' in like manner, yields to the holders of that stock the annual interest which its name imports, payable in two equal half-yearly dividends.

As the capital of any trading company is raised for a particular purpose, and limited by act of parliament to a certain amount, it follows that when that sum is completed, no more stock can be purchased of the company; though shares, already purchased, may be transferred from one person to another. "This being the case, there is often a great disproportion between the original value of the shares, and the price which is given for them when transferred.

To find the Amount of any Quantity of Stock, at a given Rate per Cent.

RULE 1. — When the rate is less than £100. Multiply the sum proposed by the given rate, and divide the product by 100.

RULE 2. — When the rate per cent. exceeds £100. Of the sum proposed take aliquot parts of £100 for the excess of the rate above £100, add the result to the given sum, and the amount will be the purchase required.

EXAMPLES FOR ILLUSTRATION.

1. What is the purchase of £1560 " 15 " 0, four per cent. consols, at £84 $\frac{1}{4}$ per cent.?

			£	s.	d.
			1560	" 15	" 0
					12
			18729	" 0	" 0
					7
$\frac{1}{4}$	$\frac{1}{4}$		131103	" 0	" 0
			390	" 3	" 9
			1314	93	" 3 " 9
				20	
			18	63	
				12	
			7	65	
				4	
			2	60	Answer, £1314 " 18 " 7 $\frac{1}{2}$ $\frac{3}{5}$.

2. What is the purchase of £1541 " 2 " 1, Bank Stock, at £226 per cent.?

			£	s.	d.
			1541	" 2	" 1
$\frac{1}{4}$	$\frac{1}{4}$		1541	" 2	" 1
			385	" 5	" 6 $\frac{1}{4}$
			15	" 8	" 2 $\frac{1}{2}$ $\frac{3}{5}$
			£3482	" 17	" 10 $\frac{3}{4}$ $\frac{3}{5}$, the answer.

EXAMPLES FOR PRACTICE.

16. What is the purchase of £1049⁶/₁₀₀0, Bank stock, at £187¹/₂ per cent.?

17. What is the purchase of £874¹³/₁₀₀6, India stock, at £168³/₄ per cent.?

18. Required the amount of £1943¹⁵/₁₀₀0, reduced £3 per cent. annuities, at £77¹/₄ per cent.?

19. What is the purchase of £3550, 4 per cent. consols, at 103³/₈ per cent.?

20. What is the purchase of £508¹⁷/₁₀₀10, old annuities, at 81⁵/₈ per cent.?

21. Bought £431¹¹/₁₀₀0, five per cent. navy annuities; what did the whole amount to, at £120⁷/₈ per cent.?

COMPOUND INTEREST.

Compound Interest, or Interest upon Interest, is that which arises, not only from the original principal, but from the interest also, as it becomes due, by adding it to the former principal, and thereby making a new and increased principal for each succeeding year.

REM. It is not lawful to receive compound interest; nevertheless, being an equitable, though an illegal, consideration, it is generally allowed in purchasing or granting annuities, leases, or reversions.

Occasional Directions for the Learner.

16. Here the excess may be divided into the following aliquot parts, viz. £50, £25, and £12¹/₂.

17. Here the excess is equal to the sum of the following parts: — £50, £12¹/₂, and £6¹/₄.

19. Multiply the rate per cent. by 35¹/₂, and the product will be the purchase required.

21. Here the excess of the rate per cent. is equal to the sum of the following aliquot parts: — £20, £¹/₂, £¹/₄, and £¹/₈. The annual interest of this description of stock has been reduced from 5 per cent. to 4 per cent.

RULE 1. — Find the interest of the given principal, for the time of the first payment, or when the first interest becomes due, and add it to the principal. This gives the amount on which the next interest is to be calculated.

2. — Find the interest of that sum, for the next payment, and its amount, as before; and proceed, in the same manner, through all the payments, and from the last result subtract the first principal, and the remainder will be the compound interest required.

QUESTION FOR ILLUSTRATION.

What will the compound interest of £550 amount to, at £5 per cent. per annum, supposing the interest, which is due at the end of every year, to be forborne during 4 years?

		£	550	
£	5	$\frac{1}{20}$	27 " 10	
			577 " 10	
	5	$\frac{1}{20}$	28 " 17 " 6	
			606 " 7 " 6	
	5	$\frac{1}{20}$	30 " 6 " $4\frac{1}{2}$	
			636 " 13 " $10\frac{1}{2}$	
	5	$\frac{1}{20}$	31 " 16 " $8\frac{1}{2}\frac{3}{20}$	
			668 " 10 " $7 - \frac{3}{20}$	the amount.
			550	given principal.
			<u>£118 " 10 " $7 - \frac{3}{20}$</u>	<u>compound interest.</u>

REM. The simple interest of the same money, at the same rate, and for the same time, is £27 " 10 " 0 $\times$ 4 = £110.

When the payments are numerous, it will be more convenient to reduce the given principal to farthings, continuing each line of the operation, where decimals occur, to two places. Thus the whole will be performed by Simple Division. Then, from the last result, as before, subtract the given principal, and the remainder will be the answer.

QUESTION FOR ILLUSTRATION.

What will the compound interest of £550 amount to, at £5 per cent. per annum, supposing the interest to be payable at the end of every half-year, but to be forborne for 8 payments, or 4 years?

First, £550 = 528 000 farthings.

Q.			
£		528000	first principal.
$2\frac{1}{2}$	$\frac{1}{40}$	13200	first half-year's interest.
		541200	second principal.
$2\frac{1}{2}$	$\frac{1}{40}$	13530	second half-year's interest.
		5 54730	third principal.
$2\frac{1}{2}$	$\frac{1}{40}$	13868.25	third half-year's interest.
		568598.25	fourth principal.
$2\frac{1}{2}$	$\frac{1}{40}$	14214.95	fourth half-year's interest.
		582813.20	fifth principal.
$2\frac{1}{2}$	$\frac{1}{40}$	14570.33	fifth half-year's interest.
		597383.53	sixth principal.
$2\frac{1}{2}$	$\frac{1}{40}$	14934.58	sixth half-year's interest.
		612318.11	seventh principal.
$2\frac{1}{2}$	$\frac{1}{40}$	15307.95	seventh half-year's interest.
		627626.06	eighth principal.
$2\frac{1}{2}$	$\frac{1}{40}$	15690.65	eighth half-year's interest.
		4) 643316.71	amount for 4 years.
		12) 160829 --	
		20) 13432 5d.	
		670 " 2 " 5 -- .71,	the amount.
		550	the first principal.
		£120 " 2 " 5 -- .71,	compound interest.

REM. In this question, the number of payments is 8, and the proportional rate per cent., for half a year, £ $2\frac{1}{2}$. It also appears, on comparing the present result with the former, that the shorter the periods of payment are, the greater will be the amount. The compound interest which accumulates from £550, when the interest is payable half-yearly, is more than what arises from the same sum, in the same time, when the interest is payable only once a year.

For short periods of time, the compound interest of any sum differs but little from the simple interest. When, however, the interest is

allowed to accumulate for any very considerable length of time, the increase, according to the method of compound interest, is very great indeed, and, in many instances, almost beyond credibility.

QUESTIONS FOR PRACTICE.

1. What is the compound interest of £500, forborne 4 years, at £5 per cent. per annum?
2. What is the compound interest of £500, (or 480 000 farthings,) forborne 4 years, at £5 per cent. per annum, supposing the interest payable half-yearly?
3. Required the compound interest of the same sum, forborne 4 years, at £5 per cent. per annum; the interest payable quarterly.
4. What is the amount of £15, 10, 0 for nine years, at £3½ per cent. per annum, compound interest?
5. What is the amount of £370, forborne 5 years, at £4 per cent. per annum, compound interest?
6. Required the amount of the same sum, forborne 4 years, at £5 per cent. per annum?

DISCOUNT.

Discount is an allowance made for the prompt payment of any sum of money, before it becomes due.

The method of computing the discount of any sum, for a given time, as invariably practised by merchants, bankers, and others, is to find the interest of the money for the time specified, and to consider *that* as the discount to be allowed; which is, in fact, always more than the true discount would come to. But it is manifestly advantageous to tradesmen, and others, to receive ready money for their goods, and to allow a liberal discount, rather than trust for payment to a future period, which

necessarily involves a greater or less degree of uncertainty in the event.

Independent of other considerations, the true present worth of any sum of money, due some time hence, is such a sum as, if put to interest, would, in the same time, and at the same rate of interest for which the discount is allowed, amount to the given sum or debt. Thus the present worth of £105, due one year hence, discounting at £5 per cent., is £100; for the amount of £100, allowing legal interest, would be, in one year, £105. Whereas the full discount of the same sum, according to the customary established mode of discounting, would be 105s., or £5⁵0, and the present worth £95¹⁵0. — In real business, therefore, it is usual to calculate the discount, at the rate of 1s. per pound per annum, or, on bills, at the rate of 1 penny per pound per month. According to this method of calculating discount, it is exactly the same as finding the interest of the money, at 5 per cent., for the given time.

The following is the method of finding the present worth and *true* discount of any sum, at a given rate per cent., by the Rule of Three.

Find the proportional interest of £100, for the given rate and time, and to the result add £100, for the amount. Then, As that amount is to £100, so is the given sum or debt to the present worth. Or, As that amount is to the first found proportional interest, so is the given sum or debt to the *true* discount.

QUESTION FOR ILLUSTRATION.

What is the *true* discount and present worth of £240, payable 3 months hence, at £5 per cent. per annum?

		£	
M.	3		5
			1 ¹ / ₄ proportional interest.
			100 assumed principal.
			<u>101 ¹/₄</u> , the amount.

$$\begin{array}{c}
 \text{As } \overset{\pounds}{101\frac{1}{4}} \text{ ————— } \overset{\pounds}{1\frac{1}{4}} \text{ ————— } \overset{\pounds}{240} \\
 \qquad \qquad \qquad 80 \\
 \frac{\cancel{4} \times \cancel{8} \times \cancel{240}}{\cancel{4}\cancel{8} \times \cancel{4} \times 1} = \pounds \frac{80}{27}
 \end{array}$$

$$\begin{array}{r}
 \pounds \\
 \left\{ \begin{array}{l} 3 \mid 80 \\ 9 \mid 26 \text{ " } 13 \text{ " } 4 \end{array} \right. \\
 \hline
 \pounds 2 \text{ " } 19 \text{ " } 3 \text{ -- } \frac{4}{9}, \text{ the true discount.}
 \end{array}$$

$$\begin{array}{r}
 \pounds \\
 240 \\
 2 \text{ " } 19 \text{ " } 3 \text{ -- } \frac{4}{9} \\
 \hline
 \pounds 237 \text{ " } 0 \text{ " } 8\frac{3}{4} \frac{5}{9}, \text{ the present worth.}
 \end{array}$$

The discount, as allowed by merchants, &c. is found as follows : —
 One penny per month is 3 pence for 3 months. Consequently,
 £ 240, at 3*d.* per pound, amount to £ 3, the discount. And £ 240,
minus £ 3 = £ 237, the present worth.

QUESTIONS FOR PRACTICE.

1. What is the common discount of £ 450, due 6 months hence, allowing £ 5 per cent. per annum?
2. Required the *true* discount of the same sum, [£ 450] for 6 months, at £ 5 per cent. per annum?
3. What is the common discount of £ 525, for 3 months, at £ 5 per cent. per annum?
4. What present money will discharge a debt of £ 300, payable in 8 months, allowing discount after the usual rate of £ 5 per cent. per annum?
5. What is the present worth of the same sum, [£ 300] due 8 months hence, allowing *true* discount for prompt payment, at the rate of £ 5 per cent. per annum?

6. Suppose I have in my possession the following notes, viz. one for £85, 10, 0, payable in 6 months, another for £355, payable in 5 months, another for £29, payable in 4 months, and a fourth for £540, payable in 3 months; what will the discount of these bills amount to, at £5 per cent. per annum?

7. Bought a quantity of goods for 1000 guineas, ready money, and sold them again for £1400, with 6 months' credit, for which I allowed the usual discount for present payment. What did I gain by this transaction?

8. What is the common allowance for discount on a bill for £1000, payable in 285 days, at £5 per cent. per annum?

9. What is the *true* discount of the bill, the particulars of which are stated in the preceding question?

10. What is the discount of a bill for £300, payable in 33 days, at £5 per cent. per annum?

11. Required the *true* discount of £300, payable 33 days hence, at £5 per cent. per annum?

Occasional Directions for the Learner.

6. Multiply the value of each bill by the time when it becomes due, add all the products together, and the sum will be the answer, in pence.

8. Multiply the value of the bill by the given number of days, and divide the product by 7300, and the quotient will be the discount in pounds. Or the discount of any sum (in pounds) for any given number of days, may be ascertained in shillings, by multiplying the given sum (in pounds) by the given number of days, and dividing the product by 365; because the interest or discount of one pound for a year, or 365 days, is one shilling.

EQUATION OF PAYMENTS.

Equation of Payments is the finding of a mean or equated time, when if a sum of money, equal to the amount of several others, due at different times, be paid, no loss will be sustained by either party, from such an arrangement, in accordance with the principles of impartial *justice*.

The Common Rule.

Multiply each payment severally by its respective time, and divide the sum of the products by the whole debt, and the quotient will be the equated time.

REM. 1. This rule is evidently adapted to the common practice of merchants and others, in discounting for prompt payment. The interest of the whole debt, for the equated time, in any of the annexed questions, is equal to the sum of the interests of each individual debt, for its respective time.

2. The following is the True Method of solution, allowing only simple interest.

Calculate the present worth of each payment, allowing *true* discount for the given time, and add the results together. Then find in what time that sum will amount to the sum of all the debts, and it will be the true equated time. — When the principal, the rate per cent., and the amount are all given, the time is found by the Rule-of-Three, thus: As the interest of the given principal for a year, is to one year; so is the difference between that principal and its amount, to the time required.

QUESTION FOR ILLUSTRATION.

If I have to pay £100 in 1 year, and £105 in 3 years; what is the equated time for the payment of the whole debt at once?

$$100 \times 1 = 100$$

$$105 \times 3 = 315$$

$$205 \overline{) 415} \quad (2\frac{1}{41} \text{ years, the answer.}$$

$$\frac{5}{205} = \frac{1}{41}$$

The *true* equated time is found as follows: — The present worth of £100, payable in 1 year, added to the present worth of £105, payable in 3 years, amount to £186 " 10 " 10 $\frac{30}{161}$. This sum is unquestionably the present worth of the two debts. Then, as the interest of £186 " 10 " 10 $\frac{30}{161}$ for a year, is to 1 year; so is £205 — £186 " 10 " 10 $\frac{30}{161}$, to 1 $\frac{882}{901}$ year, the true equated time for the payment of the whole sum at once.

QUESTIONS FOR PRACTICE.

1. There is a debt of £95 to be paid as follows, viz. £40 in 6 months, £30 in 9 months, and the rest in 10 months; what is the equated time for the payment of the whole?

2. A. owes B. £70, payable as follows, viz. £15 present, £45 in 4 months, and £10 in 6 months; what is the equated time to pay the whole?

3. A debt is to be paid as follows, viz. $\frac{1}{6}$ at 3 months, $\frac{1}{5}$ at 4 months, $\frac{1}{4}$ at 5 months, and the rest at 6 months; required the equated time to pay the whole at once?

4. A. Owes B. £240 to be paid as follows, viz. £60 in $1\frac{1}{2}$ month, £80 in $4\frac{1}{2}$ months, and the rest in $2\frac{7}{10}$ months, what is the equated time for the payment of the whole?

5. Suppose £40 are to be paid at the end of 2 years, and £210 at the end of 8 years, what is the equated time to pay the whole?

6. Suppose I have to receive £1009, 10, 0 as follows, viz. £540 in three months, £29 in 4 months, £355 in 5 months, and £85, 10, 0 in 6 months; required the equated time for the payment of the whole at once.

Occasional Directions for the Learner.

2. When any sum is due immediately, the time (or multiplier) for that sum is 0.

3. When no particular sum is mentioned in the question, it is in general, most convenient to suppose the debt £1



BARTER.

Barter is the exchanging of goods among merchants, on certain conditions, so that neither party may suffer loss ; and when the articles exchanged are not of equal value, the difference is paid in money.

REM. As it is impossible to adapt a general rule for the solution of all questions, given under the head of Barter, Occasional Directions, when they appear necessary, will be inserted at the bottom of the page, with proper numerical references to those Questions to which they respectively belong.

QUESTION FOR ILLUSTRATION.

Two merchants, A. and B., barter. A. has 1 cwt. 2 qr. $26\frac{3}{5}$ lb. of pepper, valued at 2s. 1d. per pound, for which B. is to give him in exchange, a certain quantity of cotton, at 10d. a pound. How much cotton must B. give A. for his pepper?

There are two methods of answering all questions of this description. The first is, to find the value or amount of the article whose quantity and price are both given, and then to calculate what quantity of the other commodity may be had for that money. The other method, which I prefer, both on account of its brevity, and for the facility it frequently affords of contracting the operation, is by one stating in the Rule-of-Three Inverse ; thus, As the price of any article, is to the given quantity thereof ; so is the price of any other article, inversely, to the quantity required.

s. d.	<i>Cwt. qr. lb.</i>	d.
As 2 " 1	1 " 2 " $26\frac{3}{5}$	x ϕ
12	5	2
2 s	2) 8 " 2 " 21	
5	<u><i>Cwt.</i> 4 " 1 " $10\frac{1}{2}$</u>	
	<u>the answer.</u>	
	F 2	

QUESTIONS FOR PRACTICE.

1. How much tobacco, at £2^{..}14^{..}0 per cwt., must be given in barter for 3 pipes of wine, at £42^{..}15^{..}0 per pipe?

2. How much sugar, at 10*d.* per lb., must be given in barter, for 20 cwt. of tobacco, at £3 per cwt.?

3. How much cloth, at 14*s.* 6*d.* per yard, can I have in barter, for 3 cwt. 3 qr. of sugar, at £3^{..}4^{..}0 per cwt.?

4. A merchant has 1250 yards of canvass, at 9½*d.* per yard, which he barter for serge, at 10¼*d.* per yard; how many yards must he receive?

5. A. received of B. 396 yards of cloth, in barter, for 189 gallons of brandy, at 14*s.* 8*d.* per gallon; what was the cloth per yard?

6. C. and D. barter. C. has 41 cwt. of hops, at £4^{..}18^{..}0*d.* per cwt., for which D. gives him 130 guineas

Occasional Directions for the Learner

1. As the price of the wine is to its quantity; so is the price of the tobacco to the quantity required, inversely. Call the middle term cwt., and the answer will be of the same denomination. — Questions of this nature are, in general, solved by the Rule-of-Three Inverse.

2. As the price of the sugar is to 1 lb.; so is the value of the tobacco to the quantity required in pounds. Here the proportion is direct. Reduce the answer to cwt.

3. As the price of the sugar is to 3 yds. 3 qrs.; so is the price of the cloth to the quantity, inversely.

5. As the quantity of brandy is to its price; so is the given number of yards to the price required. The proportion is inverse, because a greater quantity requires a proportional diminution in the price.

6. From the value of the hops, found either by Practice or Compound Multiplication, subtract 130 guineas, and the remainder will be the balance, for which prunes to that amount are to be received. The quantity of prunes is then ascertained by the Rule-of-Three Direct.

in part, and the rest in prunes at 4*d.* per lb.; how many cwt. of prunes must C. receive?

7. E. and F. bartered: E. gave to F. 45 gallons of brandy, at £1*7**0* per gallon, for which he received of F. £38*5**0* in money, and 500 lb. of cotton; what was the cotton per lb.?

8. G. has 1600 lb. of pepper, at 16*d.* per lb., which he barter with H. for two sorts of goods, the one at 8*d.*, the other at 5*d.* per lb., and to have of each sort an equal quantity; how many pounds of each must he receive?

PROFIT AND LOSS.

Profit and Loss, or, as it is sometimes called, Loss and Gain, is a rule by which merchants calculate their profit or loss in the purchase and sale of goods, and it instructs them how to increase or diminish the price of any article, so as to gain or lose at any given rate per cent.

REM. Questions in this rule are generally solved, either by the Rule-of-Three, or by Practice.

QUESTIONS FOR ILLUSTRATION.

1. Bought 12 tons of steel, at £69 per ton, and sold the same at 8*d.* a pound; what is the profit or loss on the sale of the whole?

Occasional Directions for the Learner.

8. As 16*d.* is to 1600 lb.; so is the sum of 8*d.* and 5*d.* to the required quantity, inversely.

First, 12 tons, at £69 per ton, amount to £828.

Also, 12 t. = 240 cwt.

240
240
240
8d. | $\frac{1}{30}$ | 26880 lb., at 8d. per lb.
£896

£.

896

828

Answer. £68 Gain.

2. Bought tea at 12s. 9d. per pound ; what must it be retailed at per pound, to gain 20 per cent.?

s. d.
20 | $\frac{1}{3}$ | 12 " 9
 2 " 6 $\frac{1}{2}$ $\frac{2}{5}$
 15 " 3 $\frac{1}{2}$ $\frac{2}{5}$ the answer.

Of course, 15s. 4d. per pound will yield something more than 20 per cent. profit ; which may be proved, and the exact rate ascertained, by the following stating :

s. d.		£.		s. d.
As 12 " 9		100		2 " 7
12				12
<u>153</u>		31		<u>31</u>

153) 3100 (20 $\frac{40}{153}$ per cent. profit, by
... 40 selling the tea at 15s. 4d.
per pound.

QUESTIONS FOR PRACTICE.

1. Bought 14 cwt. 1 qr. 15 lb. of cheese, at £2⁰₈ per cwt., and sold it again for £2¹⁵₀ per cwt.; what was the gain upon the whole?

2. If I buy 14 cwt. 1 qr. 15 lb. of cheese, for £29⁴₇, and sell it for £39¹⁰_{7 $\frac{1}{2}$} ; what do I gain per cwt.?

3. If I buy 14 cwt. 1 qr. 15 lb. of cheese, for £29⁴⁷, and sell it for £39¹⁰⁷ $\frac{1}{2}$; what do I gain per cent.?

4. If 18 cwt. of hops be bought for 100 guineas; at what rate per lb. must I sell them, to gain 16 guineas upon the whole?

5. Bought 110 tons of iron, at £17 per ton; at what rate must I sell it per cwt., to gain £7 $\frac{1}{2}$ per cent.?

6. If I buy 28 pieces of stuffs, at £4 per piece, and sell 13 of them, at £5¹⁰⁰ per piece; at what rate per piece must I sell the rest, in order to gain £12 $\frac{1}{2}$ per cent. upon the whole?

7. If a tun of wine cost £45¹⁹⁶; at what prices must I sell it to gain £10 per cent. — £15 per cent. — or £26 $\frac{1}{2}$ per cent.?

8. Bought goods for £57¹⁹²; at what price must they be sold to gain £24 per cent.?

9. If a yard of cloth cost 9s. 2d., and is sold for 7s. 9 $\frac{1}{2}$ d.? what is the loss per cent.?

10. At 2d. profit in the shilling, how much is the gain per cent.?

11. At 3 $\frac{1}{4}$ d. profit in the shilling, how much per cent.?

12. At 3s. 11 $\frac{1}{2}$ d. profit in the pound, how much per cent.?

Occasional Directions for the Learner.

7. To this question there are three different answers, adapted to the several rates per cent., given in the question.

10. Of £100 take the aliquot part of a shilling for 2d., and the result will be the gain per cent.

12. Of £100 take the aliquot parts of a pound for 3s. 4d., and 5d. and 2 $\frac{1}{2}$ d.; then the sum of the quotients will be the gain per cent. — Or thus, multiply 3s. 11 $\frac{1}{2}$ d. by the component parts of 100, and the product will be the answer.

13. If I buy $17\frac{1}{2}$ cwt. of sugar for 70 guineas, and retail it at $9\frac{3}{4}d.$ per lb. ; what shall I gain by the whole ?

14. If when I sell cloth at 7s. per yard, I gain $\pounds 2\frac{1}{4}$ per cent. ; what will be the gain per cent. when it is sold at 8s. 6d. per yard ?

15. Sold goods for 50 guineas, and lost thereby $\pounds 20$ per cent. ; whereas in trade I ought to have gained $\pounds 17$ per cent. : how much under their just value were the goods sold ?

FELLOWSHIP.

Fellowship, in general, is a rule by which merchants, trading in company with a joint stock, are enabled to ascertain each partner's share of the gain or loss, in proportion to the money advanced, and the time of its continuance.

By the same rule, also, a bankrupt's estate may be divided amongst his creditors, in proportion to their respective debts, generally according to a certain rate of poundage ; and legacies may, in like manner, be adjusted, when there is a deficiency of assets, to enable the executors to pay the debts, and the various specific sums bequeathed by the testator.

There are two kinds of Fellowship, Single and Double. Single Fellowship, or Fellowship independent of time, is when different stocks are employed for the same

Occasional Directions for the Learner.

15. Stated thus, As $\pounds 100$ — $\pounds 20$ is to $\pounds 20 + \pounds 17$; so is $\pounds 52\text{ }10\text{ }0$ to the answer.

time. Double Fellowship, or Fellowship with time, is when different stocks are employed for unequal periods of time.

SINGLE FELLOWSHIP.

RULE. — As the sum of the stocks, is to the whole gain or loss; so is each person's particular stock, to his proportional share of the gain or loss.

Or, in dividing the estate and effects of a bankrupt. As the sum of the bankrupt's debts, is to the amount of his effects; so is each creditor's particular debt, to his proportional share of the effects. In almost every case of bankruptcy, or deed of assignment, a different mode of computation generally prevails, which is, to ascertain, in the first place, from the sum of the assets, and the amount of the debts, the proportional rate per pound; and from that rate to calculate the amount of each creditor's dividend respectively. — Sometimes two or more dividends, relative to the same concern, are paid by instalments at different periods; and when the last or closing dividend is paid, it is called the *final* dividend.

Method of Proof. — Add all the shares together, and the amount will be equal to the whole gain or loss.

QUESTION FOR ILLUSTRATION.

Two merchants, A. and B., embark together in business. A. puts in, for his share of the capital, £3200, and B., £2400; they gain £1390. What is each person's share of the gain?

$$\begin{array}{r}
 \text{£.} \\
 3200, \text{ A.'s stock.} \\
 2400, \text{ B.'s stock.} \\
 \hline
 5600, \text{ Sum of the stocks.} \\
 \hline
 \text{F } 5
 \end{array}$$

Then, as $\begin{array}{ccc} \text{£.} & & \text{£.} \\ 5600 & \text{---} & 1390 \end{array}$ $\begin{array}{ccc} \text{£.} & & \text{£.} \\ & \text{---} & 3200 \end{array}$

$$\frac{1390 \times \overset{4}{88\cancel{0}\cancel{0}}}{\underset{7}{88\cancel{0}\cancel{0}}} = \frac{\overset{\text{£.}}{5560}}{7}$$

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 7) 5560 \text{ " } 0 \text{ " } 0 \\ \hline 794 \text{ " } 5 \text{ " } 8\frac{1}{2}\frac{2}{7}, \text{ A.'s share.} \end{array}$$

As, $\begin{array}{ccc} \text{£.} & & \text{£.} \\ 5600 & \text{---} & 1390 \end{array}$ $\begin{array}{ccc} \text{£.} & & \text{£.} \\ & \text{---} & 2400 \end{array}$

$$\frac{1390 \times \overset{3}{24\cancel{0}\cancel{0}}}{\underset{7}{88\cancel{0}\cancel{0}}} = \frac{4170}{7}$$

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 7) 4170 \text{ " } 0 \text{ " } 0 \\ \hline 595 \text{ " } 14 \text{ " } 3\frac{1}{4}\frac{5}{7}, \text{ B.'s share.} \end{array}$$

Proof. $\begin{array}{r} 794 \text{ " } 5 \text{ " } 8\frac{1}{2}\frac{2}{7} \\ 595 \text{ " } 14 \text{ " } 3\frac{1}{4}\frac{5}{7} \\ \hline \text{£}1390 \text{ " } 0 \text{ " } 0, \text{ the whole gain.} \end{array}$

QUESTIONS FOR PRACTICE.

1. Two persons, C. and D., trade together, with a joint stock of £350. C. put into stock, £220, and D. the remainder. Having gained by trade, £375; what is each person's share thereof?

2. Three persons, E., F., and G., trade together, with a joint stock; of which E. put in £276^s15^d0, F. £129, and G. £92^s10^d0; they gain £99^s13^d0. Required each person's share of the gain.

3. Four persons in partnership, H., K., L., and M., put into stock £270, - - £461, - - £500, and £329,

respectively, and at the end of three years they have gained £1070. What is the share of each person?

4. A bankrupt is indebted as follows, viz. to N. £827²/₀, to O. £1213, and to P. £456¹⁸/₀. His estate, worth £1947¹⁵/₀, is to be divided amongst his creditors; what is the proportional share of N.?

5. Q., R., and S., purchased a ship, of which Q. had $\frac{2}{3}$, R. $\frac{1}{8}$, and S. the remainder, for which he gave 500 guineas; required S.'s proportional share of the vessel, and the money advanced by each of the other partners.

6. A ship, worth £1200, being totally lost, of which $\frac{1}{4}$ belonged to T., $\frac{1}{6}$ to V., and the rest to W. What loss will each person sustain, supposing £850 to be insured?

DOUBLE FELLOWSHIP.

RULE. — Multiply each person's stock by the time of its continuance, and add the several products together. Then, As the sum of the products is to the whole gain or loss; so is the product of each person's stock and time, to his proportional share of the gain or loss.

REM. — When both the stocks and times are equal, the shares of each person will also be equal, and may be found by division. And when the times are equal, and the stocks unequal, the shares will be in the same proportions as the stocks, and may be ascertained by Single Fellowship. But when the stocks and times are both unequal, the shares will be as their products, and must be found by Double Fellowship.

QUESTION FOR ILLUSTRATION.

Two merchants, A. and B., lay out in trade, £5600; A. having put into stock £3200, for 8 months, and B. £2400, for 12 months. On their dissolving partnership, they find they have gained in all, £1530. What is each person's share of the profits?

First, $3200 \times 8 = 25600$ prod. of A.'s stock and time.
 $2400 \times 12 = 28800$ prod. of B.'s stock and time.
54400 sum of the products.

Then, per rule, as 54400 ————— 1530 ————— 25600

$$\begin{array}{r} 8 \\ 90 \quad 27 \\ 1888 \times 78888 = \text{£.} \\ \hline 84488 \\ 68 \\ 17 \end{array}$$

= 720, A.'s share.

As 54400 ————— 1530 ————— 28800

$$\begin{array}{r} 9 \\ 90 \quad 28 \\ 1888 \times 78888 = \text{£.} \\ \hline 84488 \\ 68 \\ 17 \end{array}$$

= 810, B.'s share.

£.
 Proof. 720
 810

£1530, the whole gain.

QUESTIONS FOR PRACTICE.

1. Three merchants, C., D., and E., enter into partnership ; C. put into stock £540, for 8 months, D. £670,

Occasional Directions for the Learner.

1. Here the product of each person's stock and time may be found and subsequently abbreviated, as follows : —

C. $540 \times 8 = 4320$, or 216, or 72
 D. $670 \times 12 = 8040$, or 402, or 134
 E. $730 \times 6 = 4380$, or 219, or 73
 Sum of the abbreviated products, 279

Then, As that sum is to the whole gain, so is 72, to C.'s share. And so, also, are 134, and 73, to the share of D., and E., respectively.

for 12 months, and E. £730, for 6 months. Having gained £550, what is each person's share of the gain?

2. Three men, F., G., and H., hold a pasture in common, for which they pay £90. F. had 8 oxen for 3 months, G. 4 oxen for 12 months, and H. 6 oxen for 5 months; required each man's share of the rent.

3. A ship's company take a prize of £1000, which they agree to divide among them according to their pay, and the time they have been on board: now the officers and midshipmen have been on board 6 months, and the sailors 3 months; the officers have 40s. a month, the midshipmen 30s., and the sailors 22s. a month; moreover, there are 4 officers, 12 midshipmen, and 110 sailors. What will each person's share of the prize come to?

4. Three merchants, X., Y., and Z., enter into partnership for 12 months; X. put into stock at first £1000, and at the end of 8 months, he advanced £500 more. Y. put in at first £1500, and at 6 months' end, he took out £375. Z. put in £2800, and at 3 months' end he took out £600, and 5 months afterwards he put in £1380. Now at the expiration of the time, they find there has been gained £792.10.0. How must this be divided amongst the partners?

Occasional Directions for the Learner

3. Multiply the pay, time, and number of men in each company continually together. Then, As the sum of the products, is to the whole prize-money; so is the product of each person's pay and time only, to his proportional share of the money. — *Method of Proof.* Multiply the share of each individual, by the number of officers, midshipmen, and sailors, respectively, and the sum of the products will be equal to the whole prize-money.

4. The method of finding the products in this Question may be exemplified in determining that of one of the partners; suppose X.

$$1000 \times 8 = 8000$$

$$\underline{500}$$

$$1500 \times 4 = 6000$$

$$\underline{\underline{14000,}}$$

Total product of X.'s stock
and time.

ALLIGATION.

Alligation is the method of compounding or mixing together two or more articles of different qualities and values, so that the whole composition may be of a mean quality and proportional value.

This Rule is divided into two principal Cases, called Alligation Medial, and Alligation Alternate. To these are usually added two supplementary Cases or Varieties, which are distinguished by the names Total and Partial ; in the former, the whole composition is limited to a given total or amount, and in the latter, one of the ingredients only is limited to a particular quantity.

CASE I. ALLIGATION MEDIAL.

Having the Quantities and Prices of the various Articles given, to find the mean Rate or Price of the whole Composition.

RULE. — Multiply each rate by its respective quantity ; then divide the sum of the products by the sum of the quantities, and the quotient will be the rate required.

QUESTION FOR ILLUSTRATION.

A tea-dealer mixes 24 pounds of tea, at 3s. per pound, with 12 pounds at 6s. per pound, 10 pounds at 10s. per pound, and 20 pounds at 12s. per pound ; how must he sell the whole per pound, so that he may neither lose nor gain by the mixture.

				<i>s.</i>
3 <i>s.</i>	×	24	=	72
6 <i>s.</i>	×	12	=	72
10 <i>s.</i>	×	10	=	100
12 <i>s.</i>	×	20	=	240
		66	{	11 484
				6 44
				7 <i>s.</i> 4 <i>d.</i> per pound.

QUESTIONS FOR PRACTICE.

1. A farmer mixes 15 bushels of wheat, at 5*s.* per bushel, 27 bushels of rye, at 3*s.* per bushel, and 30 bushels of barley, at 2*s.* per bushel; what is a bushel of this composition worth?

2. Suppose 7 $\frac{1}{2}$ lb. of tea, at 5*s.* 8*d.* per pound, 9 lb., at 8*s.* per pound, and 21 $\frac{3}{4}$ lb., at 12*s.* per pound, were mixed together; what is a pound of it worth?

3. Mixed 20 gallons of rum, at 9*s.* per gallon, 35 gallons, at 9*s.* 6*d.* per gallon, with 48 $\frac{3}{4}$ gallons, at half-a guinea per gallon, and 10 gallons of water together; what is a gallon of this composition worth?

4. Made a composition of 9 gallons of spirits, at 12*s.* per gallon, 9 gallons of wine, at 5*s.* 3*d.* per gallon, and 6 gallons of cider, at 1*s.* 3*d.* per gallon; required the price of a gallon of this mixture.

5. Having mixed together 21 gallons of ale at 1*s.* 2*d.* per gallon, 17 gallons, at 1*s.* 5 $\frac{1}{2}$ *d.* per gallon, 13 gallons, at 1*s.* 6*d.* per gallon, and 5 gallons, at 2*s.* per gallon; what is the value of a gallon of this mixture?

Occasional Directions for the Learner.

3. If any one article be of little or no value with respect to the rest, as water mixed with spirits, alloy with gold, &c. its value is supposed to be nothing.

6. A goldsmith melted 12 ounces of gold, at £4 per ounce, 3 ounces, at £4⁶/₈ per ounce, and 9 ounces, at £4¹³/₄ per ounce; what is a pound of this composition worth?

7. A refiner melts 10 lb. of gold of 20 carats fine, 16 lb. of 18 carats fine, and 42 lb. of pure gold; how many carats fine is the whole composition?

8. A goldsmith melts 8 lb. 6 oz. of gold of 14 carats fine, and 6 lb. 8 oz. of 16 carats fine, with 3 lb. of alloy; how many carats fine is this mixture?

CASE II. ALLIGATION ALTERNATE.

Alligation Alternate is the reverse of Alligation Medial, and shews how to find what quantity of any number of articles, whose rates are given, will make a composition of a given rate or value.

RULE 1. — Write the rates of the articles in a column under one another, and place the mean rate, in the same denomination as the given rates, on the left hand.

2. Connect the rate of each article which is less than the mean rate, with all those that are greater, and place the differences *alternately*, opposite the rates with which they are respectively linked. Then,

Occasional Directions for the Learner.

6. Having found the price of an ounce, multiply it by 12, and the product will be the answer to the question.

7. If 1 lb., or any other quantity of gold, be divided into 24 equal parts, these parts are denominated carats; but pure gold is generally mixed with some baser metal, which is termed the alloy, and the composition is said to be of so many carats fine, according to the proportion of pure gold which it contains: thus, if 20 carats of pure gold and 4 carats of alloy are mixed together, it is said to be 20 carats fine.

3. If only one difference stand against any rate, it will be the quantity belonging to that rate; and if there be two or more differences, their sum will be the quantity required.

REM. — The resulting quantities may frequently be abbreviated, by dividing each of them by some common measure.

QUESTIONS FOR ILLUSTRATION.

1. How much wine at 4*s.*, 6*s.*, 9*s.*, and 12*s.* per gallon must be mixed together, that the whole composition may be worth 8*s.* per gallon?

	<i>s.</i>		<i>gal.</i>		<i>s.</i>
	4		1 + 4 = 5	at	4
<i>s.</i>	6		1 + 4 = 5	at	6
8	9		4 + 2 = 6	at	9
	12		4 + 2 = 6	at	12

	<i>s.</i>		<i>s.</i>
Proof.	4 × 5 =		20
	6 × 5 =		30
	9 × 6 =		54
	12 × 6 =		72
		22 { 2	176 <i>s.</i>
		11	88
			8 <i>s.</i> per gallon.

2. A tea-dealer would mix four sorts of tea together, namely, at 7*s.*, 5*s.* 6*d.*, 5*s.*, and 4*s.* 6*d.* per pound; what proportion of each must he use, in order that he may sell the whole at 6*s.* per pound?

	<i>sixpences.</i>		<i>lb.</i>	<i>s.</i>	<i>d.</i>
	14		1 + 2 + 3 = 6,	or 3	at 7 " 0
<i>sixp.</i>	11		2.....2,	or 1	at 5 " 6
12	10		2.....2,	or 1	at 5 " 0
	9		2.....2,	or 1	at 4 " 6
				6	at 6 " 0

QUESTIONS FOR PRACTICE.

9. It is required to mix together brandy at 15s. per gallon, wine at 6s. per gallon, cider at 1s. per gallon, and water, in such proportion that the mixture may be worth 5s. per gallon.

10. A goldsmith has gold of 24, 22, 18, and 17 carats fine; how much of each must he take, to make a composition of 21 carats fine?

 VARIETY I. ALLIGATION TOTAL.

When the whole Composition is limited to a certain given Amount.

RULE. — Find the several quantities, as before, by the preceding Case, and add the results together, to ascertain their amount. Divide the given quantity by that amount, and multiply each of the quantities, first found, by the quotient, and the amount of those products will be equal to the amount given in the question.

QUESTION FOR ILLUSTRATION.

How many gallons of water must be mixed with three different sorts of rum, at 12s., 15s., and 18s., per gallon, so as to fill a vessel containing 294 gallons, and that the whole composition may be worth 14s. per gallon:

	s.		
	18		$2 + 14 = 16$
	15		$2 + 14 = 16$
s.	12		$4 + 1 = 5$
14	0		$4 + 1 = 5$
			<u>42</u> , the amount.

Then, $294 \div 42 = 7$ the quotient.

		<i>gal.</i>		<i>s.</i>		<i>s.</i>
16	×	7	=	112	at	18, = 2016
16	×	7	=	112	at	15, = 1680
5	×	7	=	35	at	12, = 420
5	×	7	=	35	of water,	= 0
				294	at	14 = <u><u>4116</u></u>

QUESTIONS FOR PRACTICE.

11. A grocer has raisins at $4d.$, $6d.$, $9d.$, and $11d.$ per pound, and he would make a mixture of 210 lb., so that a pound of it might be worth $8d.$ How much of each sort must he take?

12. How much gold of 15, 17, 18, 20, and 22 carats fine, must be mixed together, to form a composition of 50 ounces, of 20 carats fine?

VARIETY II. ALLIGATION PARTIAL.

When one of the Ingredients only is limited to a certain given Quantity.

RULE. — Find the several quantities, as before, by the Rule of Alligation Alternate. Divide the given quantity of the limited ingredient by the quantity which is opposite to that ingredient, and multiply each of the quantities, first

Occasional Directions for the Learner.

12. When one of the ingredients is the same as the mean rate, the quantity of that ingredient may be taken *ad libitum*, as it will neither increase nor diminish the average price of the whole. Suppose, therefore, one fifth of the whole composition, or 10 ounces, to be gold of 20 carats fine, and there will remain 40 ounces for the sum of the rest. Then proceed as before, according to the rule.

found, by the quotient, and the several products will be the quantities required.

QUESTION FOR ILLUSTRATION.

How much wine at 4s., 5s., 6s., and 7s. per gallon, must be mixed with *18 gallons at *5s. 6d. per gallon, so that the whole mixture may be worth 5s. 3d. per gallon?

<i>Three-pences.</i>			
<i>Three-pences.</i> 21	16	1 + 3 + 7 = 11	
	20	1 + 3 + 7 = 11	
	*22	5 + 1 = *6	
	24	5 + 1 = 6	
	28	5 + 1 = 6	

$18 \div *6 = 3$ the quotient or multiplier.

	<i>gal.</i>		<i>shillings.</i>
11 x 3 =	33	at 4s.	= 132
11 x 3 =	33	at 5s.	= 165
*6 x 3 =	*18	at *5s. 6d.	= 99
6 x 3 =	18	at 6s.	= 108
6 x 3 =	18	at 7s.	= 126
	<u>120</u>	at 5s. 3d.	= <u>630</u>

EXAMPLES FOR PRACTICE.

13. How much gold of 15, 17, and 18 carats fine, must be mixed with 75 ounces of 22 carats fine, so that the whole composition may be 20 carats fine?

14. A grocer would mix tea at 8s., 10s., and 12s., with 20 lb. at 6s. per lb.; how much of each sort must he take, so that the whole may be sold at 8s. per pound?

EXCHANGE.

Exchange is the paying or receiving of money in one country, for its equivalent value in the money of another country, according to a given rate, or course of exchange.

Bills of Exchange, or drafts, are written orders on bankers and others, for the payment of certain specified sums of money at an appointed time.*

* The person who draws a bill of exchange, is called the *drawer*; and the person to whom it is addressed, the *drawee*, who is afterwards called the *acceptor*, if he pledges himself for its payment when it becomes due. The person in whose favour the bill is drawn, is called the *payee*.

For example, in the annexed copy of a bill of exchange, J. Robinson is the *drawer*, J. Birchall the *payee*, and the firm of Jones, Lloyd, and Co., the *drawee*, and subsequent *acceptor*.

(COPY.)

No. 573. £500 " 0 " 0.

Manchester, July 16, 1838.

*Two months after date, pay to Mr. James Birchall, or order,
Five Hundred Pounds. Value received.*

To

John Robinson.

Messrs. Jones, Lloyd, and Co.,

Bankers,

London.

When the holder or possessor of a bill disposes of it, either in the way of negotiation or payment, he writes his name on the back of it, which is called indorsing; and every indorser is an additional security for the payment. The payee, or his procurator on his account, is always the first indorser; and when any particular indorsement is in favour of a certain individual person, or company, it is called a special

The course or rate of exchange, is such a variable sum of the money of one country, as is given for a certain stated piece or sum of money of another country, which fluctuates, according to the demand for bills, or to the circumstances of trade.

The par of exchange is the intrinsic value of the money of one country, compared with a certain piece or sum of money of another country, both with respect to its weight and fineness.* When the course of exchange is above par, England gains by the exchange, and *vice versâ*, loses when it is below par.

The term or duration of a bill of exchange varies according to the agreement of the parties, or the custom of different countries. Some bills are drawn payable on demand or at sight; others, at a certain number of days

indorsement, and the person to whom it is thus made payable, is called the *indorsee*.

When a bill of exchange is presented for acceptance, it is generally left till the next day, and any writing, either by the drawee or his clerk, which does not imply a refusal, is deemed a legal acceptance.

If a bill of exchange be refused acceptance, it is immediately put into the hands of a notary public, and noted for non-acceptance.

If an accepted bill be afterwards refused payment, it is noted or protested accordingly, and returned to any one of the preceding indorsers, or to the drawer, by which the drawer, or any of the indorsers, at the option of the holder, is liable to pay the value of the bill, together with all the expences which have been incurred upon it; but if the holder or possessor of the bill make any unnecessary delay in returning it, he can, in that case, sue the acceptor only.

* By fineness is to be here understood, the proportion of pure gold or silver, and of copper or alloy, in any given species of coin. In Great Britain, the standard quality or fineness of gold coin is 22 carats of pure gold, and 2 carats of copper or alloy, the carat being one twenty-fourth part of the whole weight. In like manner, the standard quality or fineness of silver coin is 11 oz. 2 dwt. of pure silver, and 18 dwt. of alloy, making together 12 ounces or 1 pound troy. The word sterling is now applied to all lawful money, current in Great Britain.

or months* (including the customary days of grace) after date; and some are drawn at usance.

Days of grace are a certain number of days, which are understood, though not expressed, to be allowed for payment, after the specified term of the bill is fully expired. Three days are allowed in the United Kingdoms of Great Britain and Ireland, except for bills payable at sight or on demand, which must be paid when presented. When a bill becomes due on a Sunday, it must be paid on the day preceding, that is, on the Saturday.

Usance is the usual term of foreign bills of exchange, between certain places, such as one, two, or three months; and double or half usance means double or half the usual term allowed between those places.

In some foreign places, there are two sorts of money, one called Banco, and the other currency. Banco bears a considerable premium, which is called agio, being the difference between £100 banco, and the money to which the same £100 is equal, when expressed in currency. Bills of exchange are always supposed to be valued and paid in banco.

The denominations of money which occur in the calculation of exchanges, are often imaginary, or merely nominal, and in most places they are also different from those in which their accounts are kept. The current money of England formerly consisted of guineas, half-guineas, and seven-shilling-pieces, in gold; these, however, have lately been superseded by a new gold coinage, in which there is no alteration, either in the comparative weight or absolute fineness, from former coinages; the

* When the term of a bill or draft is expressed in months, calendar months are always understood. Thus, for example, if a bill at two months be dated January 2, the term of two months will expire on the 2d of March, and the bill, of course, must be paid on the 5th of March.

sovereign, or 20-shilling piece, being 20 twenty-one parts of the weight and value of a guinea. The half-sovereign, as its name imports, is half the value of the sovereign, or 10 shillings. — The new silver coinage is of the old standard fineness, of 11 oz. 2 dwt. of pure silver to 18 dwt. of alloy; but 1 lb. troy, of this standard, is now coined into 66 shillings, instead of 62 shillings, as was formerly the case. The crown, or 5-shilling piece, weighs 18 dwt. $4\frac{4}{11}$ gr.; the half-crown, 9 dwt. $2\frac{2}{11}$ gr.; the shilling, 3 dwt. $15\frac{3}{11}$ gr.; and the sixpence, 1 dwt. $19\frac{7}{11}$ gr. The value of a lb. of gold is $46\frac{2}{4}\frac{9}{0}$ sovereigns, or £46¹⁴6; and the value of a lb. of silver, 62 shillings, or £3²0. — The copper pieces, now in circulation, consist, as formerly, of pennies, half-pennies, and farthings; but the farthing, owing to its diminutive value, is seldom used.

IRELAND.

Accounts are kept in Ireland, in pounds, shillings, and pence, as in England. The par of exchange is one shilling and one penny (or 13*d.*) Irish, for one shilling English; consequently, 100 pounds English are equal to £108⁶8 (or £108 $\frac{1}{3}$) Irish.

The course of exchange varies from £106 to £120 Irish per £100 English.

QUESTION FOR ILLUSTRATION.

Reduce £486¹⁵0 English, into Irish money, the course of exchange being £112 Irish, for £100 English.

		£.	s.	d.
$\left \begin{array}{c} \text{£.} \\ 10 \\ 2 \end{array} \right $	$\frac{1}{10}$	486	15	0
		48	13	6
		9	14	$8\frac{1}{4}\frac{3}{5}$
		<u>£545³2$\frac{1}{4}\frac{3}{5}$</u>		

QUESTIONS FOR PRACTICE.

1. Reduce £525 English, into its equivalent Irish money, the course of exchange being at par, or £108 $\frac{1}{3}$ Irish, for £100 English.

2. Reduce £876 „ 12 „ 6 Irish, into its equivalent English money, the exchange being at £112 $\frac{1}{4}$ Irish per £100 English.

3. London remits to Dublin, £787 „ 15 „ 0 sterling; how much Irish money must be received for it, exchange at £111 $\frac{5}{8}$ Irish per cent?

4. Dublin remits to London, £450 „ 7 „ 6 Irish; for how much sterling must Dublin be credited, exchange at £112 Irish, per £100 sterling?

5. London remits to Dublin £787 „ 15 „ 0 sterling; how much money must be received for it at Dublin, exchange at £110 $\frac{1}{2}$ Irish, for £100 English?

6. London remits to Dublin, £469 „ 17 „ 0 sterling; how much must be received for it in Ireland, when the course of exchange is £109 $\frac{7}{8}$ Irish, for £100 English?

Occasional Directions for the Learner.

1. Of the given sum (£525) take the aliquot part of £100, for £8 $\frac{1}{3}$, (being $\frac{1}{12}$ of £100) and add the result.

2. By the Rule-of-Three direct, thus; As £112 $\frac{1}{4}$ is to £100; so is £876 $\frac{5}{8}$ to the answer. — The proof of this sum, by practice, will be no bad exercise for the Learner.

4. As £112 is to £100; so is £450.375 to the answer. Remove the decimal point in the third term, two places to the right hand, and divide the result (£45037 „ 10 „ 0) by the component parts of the first term.

5. This question is taken from Mr. Bonnycastle's Scholar's Guide to Arithmetic, the third edition, page 219; where an erroneous answer is subjoined, viz. £791 „ 13 „ 3 $\frac{1}{4}$ $\frac{1}{5}$.

6. Multiply the given sum, £469 „ 17 „ 0, by £9 $\frac{7}{8}$ (or £10 minus £ $\frac{1}{8}$), and divide the product by 100. Add the result to £469 „ 17 „ 0, and the sum will be the answer.

NORTH AMERICA, AND THE WEST INDIES.

In America and the West Indies, accounts are kept in pounds, shillings, and pence, as in Great Britain; but as the money they make use of for carrying on their trade consists chiefly of foreign coins and paper currency, which are subject to many disadvantages, it is generally at a considerable discount.

QUESTIONS FOR ILLUSTRATION.

1. Reduce £845 " 17 " 6 currency, into sterling, the exchange being £180 currency, for £100 sterling.

$$\begin{array}{rcccl} \text{£.} & & \text{£.} & & \text{£.} \quad \text{s.} \quad \text{d.} \\ \text{As } 180 & \text{---} & 100 & \text{---} & 845 \text{ " } 17 \text{ " } 6 \end{array}$$

$$\begin{array}{r} \phantom{845 \text{ " } 17 \text{ " } 6} 5 \\ \hline 845 \text{ " } 17 \text{ " } 6 \times 180 \\ \hline 180 \\ 9 \\ \hline \text{£.} \quad \text{s.} \quad \text{d.} \\ 845 \text{ " } 17 \text{ " } 6 \\ \phantom{845 \text{ " } 17 \text{ " } 6} 5 \\ \hline 9 \overline{) 4229 \text{ " } 7 \text{ " } 6} \\ \underline{\text{£}469 \text{ " } 18 \text{ " } 7 \frac{1}{4} \frac{1}{3}} \end{array}$$

2. London draws upon Port-Royal, in Jamaica, for £472 " 18 " 6 sterling; how much currency will this amount to, when the exchange is £140 currency, per £100 sterling?

$$\begin{array}{r|l|l} \text{£} & & \text{£.} \quad \text{s.} \quad \text{d.} \\ \hline 20 & \frac{1}{5} & 472 \text{ " } 18 \text{ " } 6 \\ 20 & \frac{1}{5} & 94 \text{ " } 11 \text{ " } 8 \frac{1}{4} \frac{3}{5} \\ & & 94 \text{ " } 11 \text{ " } 8 \frac{1}{4} \frac{3}{5} \\ \hline & & \underline{\text{£}662 \text{ " } 1 \text{ " } 10 \frac{3}{4} \frac{1}{5}} \end{array}$$

QUESTIONS FOR PRACTICE.

7. To how much currency will £296^{..}15^{..}8 sterling amount, when the course of exchange is £165½ currency, per cent. sterling?

8. Reduce £917^{..}18^{..}0 sterling into currency, the course of exchange being £175 currency, per £100 sterling.

9. Barbadoes is indebted to London, £1464^{..}13^{..}9 currency; what sterling must be remitted, when the exchange is £135 currency, per cent. sterling?

10. London consigns goods to Virginia, amounting to £579^{..}19^{..}6 sterling, which are sold for £847^{..}15^{..}6 currency; what sterling must the factor remit, allowing £5 per cent. for commission and charges, the exchange being £130 currency, per cent. sterling?

PORTUGAL.

Accounts are kept at Lisbon, in milrees, and rees, which are commonly distinguished by a mark placed between them; thus 355∞728, that is, 355 milrees, and 728 rees. The course of exchange is from 60 to 67 pence sterling, per milree.

THE TABLE.

	s.	d.
1000 Rees, or 2½ Crusadoes, make 1 Milree.	[5	7½
400 Rees, - - - - - 1 Crusado.	[2	3

Occasional Directions for the Learner.

8. The answer to this question may be proved by reversing the operation, saying, by the Rule-of-Three, As £175 is to £100; so is the answer to the sterling money given in the question.

10. Find the factor's commission on £847^{..}15^{..}6, by taking £5 an aliquot part of £100, subtract the result, and, by the Rule-of-Three, as in the preceding question, find how much sterling the remainder is equal to, according to the given rate of exchange.

QUESTION FOR ILLUSTRATION.

What is the value of 1548 milrees, 500 rees, exchange at 5s. $7\frac{1}{2}d.$ sterling per milree?

			1548	at 5s. $7\frac{1}{2}d.$
	5s.	$\frac{1}{4}$	387	" 0 " 0
	$7\frac{1}{2}d.$	$\frac{1}{8}$	48	" 7 " 6
(of a milree)	500 r.	$\frac{1}{2}$		2 " 9 $\frac{3}{4}$
			<u>£435</u>	<u>" 10 " 3 $\frac{3}{4}$</u>

QUESTIONS FOR PRACTICE.

11. What is the amount of 871 milrees, exchange at 6s. 3d. per milree?

12. In 892 mil. 96 rees, how many pounds sterling, exchange at 5s. 7d. per milree?

13. In 2716 crusadoes, how much sterling, exchange at 5s. $2\frac{5}{8}d.$ sterling per milree?

14. In £503 " 5 " 8 sterling, how many crusadoes, exchange at 5s. $4\frac{1}{2}d.$ per milree?

SPAIN.

Accounts are kept at Madrid, Cadiz, and Bilboa, principally in piastres, or pieces-of-eight, rials, and maravedies; and the exchange is from 35 to 45 pence sterling per piastre. The piastre, or pezza, is sometimes called the dollar of exchange.

The money of Spain is distinguished into Plate and Vellon, and 32 rials Vellon are equal to 17 rials of Plate. The latter only is used in the exchange with England.

The usance is 60 days after date, with 6 days of grace.

Occasional Directions for the Learner.

14. Stated thus, As 5s. $4\frac{1}{2}d.$ is to $2\frac{1}{2}$ crusadoes; so is the given sterling to the answer. — The three preceding questions, viz. 11, 12, and 13, are answered by the rule of Practice.

THE TABLE.

8 Rials	- - - -	make 1 Piastre.	<i>s. d.</i> [3 " 7
34 Maravedies	- - —	1 Rial.	[0 " 5 $\frac{1}{4}$ $\frac{1}{2}$

EXAMPLE FOR ILLUSTRATION.

Reduce 2375 piastres, 6 rials, 17 maravedies, into sterling, at 38*d.* sterling per piastre.

By the Rule-of-Three.

<i>pi.</i>	<i>d.</i>	<i>pi.</i>	<i>r.</i>	<i>m.</i>
As 1	38	2375	6	17
8		8		
<u>8</u>		19006		
2		2		
<u>16</u>		38013	half rials.	

$$\begin{array}{r}
 19 \\
 38013 \times \cancel{38} \\
 \hline
 \cancel{38} \\
 8 \\
 \hline
 38013 \\
 19 \\
 \hline
 8 \overline{) 722247} \\
 12 \overline{) 90280} \frac{31}{42} \\
 20 \overline{) 7523} \text{ " } 4 \\
 \hline
 \underline{\underline{\pounds 376 \text{ " } 3 \text{ " } 4 \frac{3}{4} \frac{1}{2}}}
 \end{array}$$

The same, by Practice.

<i>s. d.</i>		<i>pi. r. m. s. d.</i>
2 " 6	$\frac{1}{8}$	2375 (6 " 17) at 3 " 2
8	$\frac{1}{30}$	296 " 17 " 6
4 r.	$\frac{1}{2}$	79 " 3 " 4
2 r.	$\frac{1}{2}$	0 " 1 " 7
17 m.	$\frac{1}{4}$	0 " 0 " 9 $\frac{1}{2}$
		0 " 0 " 2 $\frac{1}{4}$ $\frac{1}{2}$
		<u><u>\pounds 376 " 3 " 4 $\frac{3}{4}$ $\frac{1}{2}$</u></u>
		G 3

EXAMPLES FOR PRACTICE.

15. How many piastres shall I receive at Madrid for £595 sterling, exchange at $41\frac{7}{8}d.$ per piastre?

16. In £125⁷/₄ sterling, how many piastres, exchange at $39\frac{1}{2}d.$ sterling per piastre?

17. Spain draws upon London for 2856 piastres; to how much sterling will the draft amount, exchange at $40\frac{1}{8}d.$ sterling per piastre?

18. If I, intending to purchase goods at Cadiz, pay into a Spanish house, £1000, for that purpose; how many piastres may I expect, as an equivalent for this money, the course of exchange being 3s. 6d. per piastre?

19. Bought goods in Spain, to the value of 4275 piastres, 5 rials, exchange at $40\frac{7}{8}d.$ sterling per piastre; how many pounds sterling must they be sold for in England, to gain £20 per cent.

FRANCE.

In France, accounts are kept either in livres, sous, and deniers; or in francs, décimes, and cents.

London exchanges with Paris at a certain number of pence sterling, from 30 to 32, for one ecu; or to receive a certain number of francs and cents, for one pound sterling.

Occasional Directions for the Learner.

19. Find, by the rule of Practice, the amount of 4275 pi. 5 rials, at $40\frac{3}{4}d.$ $\frac{1}{2}q.$ Then of the result take £20 an aliquot part of £100, and add.

THE TABLE.

8 Ecus - - - - -	make 1 Louis d'or.	s. d. [20 " 0
3 Livres Tournois - - - - -	1 Ecu.	[2 " 6
20 Sous or Sols - - - - -	1 Livre.	[0 " 10
12 Deniers - - - - -	1 Sou.	[0 " 0 $\frac{1}{2}$
10 Décimes, or 100 Cents. -	make 1 Franc.	[0 " 10 $\frac{1}{8}$
10 Cents, or Centimes - -	1 Décime.	[0 " 1 $\frac{1}{30}$

* * * The franc, or new livre, is $1\frac{1}{4}$ per centum better than the old livre tournois, 80 francs being equal in value to 81 livres. The term tournois, in France, is of the same import as the term sterling, in England.

To reduce any number of Livres, Sous, and Deniers, into Pounds Sterling, at a given Rate per Ecu.

RULE.—Divide the given sum (considered as so many pounds, shillings, and pence) by 3, and then take the proper aliquot part or parts of a pound for the given rate, and the result will be the answer in pounds sterling.

QUESTIONS FOR ILLUSTRATION.

1. Reduce 2627 livres tournois, 17 sous, 6 deniers, into pounds sterling, exchange at $31\frac{1}{2}d.$ per ecu.

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 3) \quad 2627 \text{ " } 17 \text{ " } 6 \\
 \hline
 875 \text{ " } 19 \text{ " } 2 \quad 2s. \ 7 \frac{1}{2}d. \\
 109 \text{ " } 9 \text{ " } 10 \frac{3}{4} \\
 5 \text{ " } 9 \text{ " } 5 \frac{3}{4} \frac{15}{20} \\
 \hline
 \text{£}114 \text{ " } 19 \text{ " } 4 \frac{1}{2} \frac{3}{4} \text{ the answer.}
 \end{array}$$

2. Reduce £875 " 17 " 6 sterling, into Francs and Cents, or into Francs, Décimes, and Cents., exchange at 23 Francs, 13 Cents, per £1 sterling.

By the Rule-of-Three, decimally.

£.	Francs.	£.
As 1	23.13	875.875
		23.13
		11386375
		2627625
		1751750
		20258.98,875

Francs. Cents.

Answer, 20258 " 98 $\frac{7}{8}$ [$.875 = \frac{875}{1000} = \frac{7000}{8000} = \frac{7}{8}$.]

Or thus, in whole numbers.

£.	Cents.	£. s. d.
As 1	2313	875 " 17 " 6
8	7007	8
8	16191	7007 half-crowns.

	16191
8	16207191
10	2025898 $\frac{7}{8}$
10	202589 " 8
	Fr. 20258 " 9 " 8 $\frac{7}{8}$

Francs. Décimes. Cents.

Answer, 20258 " 9 " 8 $\frac{7}{8}$

QUESTIONS FOR PRACTICE.

20. Reduce 23554 livres, 15 sous, into pounds sterling, exchange at $31\frac{3}{8}d.$ sterling per ecu.

21. In £314 " 11 " $7\frac{1}{4}$ sterling, how many livres, exchange at $30\frac{3}{8}d.$ per ecu?

22. What is the value of 750 Louis d'ors, exchange at $30\frac{1}{8}d.$ sterling per ecu?

23. To how much sterling will 15 358 liv. 14 sous, 9 den. amount, exchange at $31\frac{5}{8}d.$ sterling per ecu?

Occasional Directions for the Learner.

23. Divide the given sum by 3, and of the quotient take the aliquot parts for $31\frac{5}{8}d.$ as follows: $30d.$ — $1\frac{1}{2}d.$ — and $\frac{1}{8}d.$

24. How many livres, sous, and deniers, are equal to 197 francs, 7 décimes, and 5 centimes?

ITALY.

Accounts are kept at Genoa, Leghorn, and Venice, in lire, soldi, and denari; and the exchange with London is from 45 to 54 pence sterling per piastre, ducat, or dollar.

THE TABLE.

	s.	d.
12 Denari - - - - - make 1 Soldo.	[0 "	0 $\frac{43}{100}$
20 Soldi - - - - - — 1 Lira.	[0 "	8 $\frac{3}{5}$
5 $\frac{3}{4}$ Lire, or 115 soldi - - — 1 Piastre or Dollar.	[4 "	1 $\frac{1}{2}$

QUESTION FOR ILLUSTRATION.

What is the amount of 5230 lire, 17 soldi, exchange at 49 $\frac{1}{2}$ d. sterling per piastre?

<i>Soldi.</i>	<i>d.</i>	<i>Lire.</i>	<i>Sol.</i>
As 115 ———	49 $\frac{1}{2}$ ———	5230 " 17	
	4	20	
	<u>198</u>	<u>104617</u>	
		200— 2 times the multiplicand.	
		<u>20923400</u>	
		209234	
		115) 20714166 (	4 180123 $\frac{21}{115}$
		.921	12 45030 $\frac{3}{4}$
		.. 141	20 3752 " 6
		266	<u>£187 " 12 " 6 $\frac{3}{4}$ $\frac{21}{115}$</u>
		366	
		21	
		Answer, £187 " 12 " 6 $\frac{3}{4}$ $\frac{21}{115}$	

Occasional Directions for the Learner.

24. As 80 francs is to 81 livres; so is the given number of francs (197.75) to the answer.

QUESTIONS FOR PRACTICE.

25. What is the value of 1876 piastres, 12 soldi, 6 denari, at $50\frac{1}{4}$ pence sterling per piastre?

26. How many lire, soldi, and denari, must be received at Leghorn, for £705 " 16 " 4 sterling, exchange at $51\frac{1}{2}$ pence sterling per piastre?

AMSTERDAM.

There are two sorts of money in Holland, called banco and currency; the former bearing a premium, which is called the *agio*.—All bills of exchange are valued and paid in banco.

Accounts are here kept in guilders, stivers, and pennings; also in Flemish pounds, shillings, and pence.

THE TABLE.

		s.	d.
6	Guilders or Florins make 1 Pound Flemish.	[10 "	6
$2\frac{1}{2}$	Guilders - - - — 1 Rix Dollar.	[4 "	$4\frac{1}{2}$
20	Stivers - - - — 1 Guilder.	[1 "	9
16	Pennings - - - — 1 Stiver.	[0 "	$1\frac{1}{20}$
20	Shillings, Flemish - make 1 Pound Flemish.		
12	Pence, or 6 Stivers — 1 Shilling.	[0 "	$6\frac{1}{4}\frac{1}{5}$
8	Pennings - - - — 1 Penny.	[0 "	$0\frac{1}{2}\frac{1}{10}$

Occasional Directions for the Learner.

25. Find the amount of 1876 piastres, at 4s. $2\frac{1}{4}$ d. per piastre, by Practice, and to the result add the value of $12\frac{1}{2}$ soldi, which may be ascertained by the Rule of-Three. — Proof by the Rule-of-Three:

As 115 soldi is to $50\frac{1}{4}$ d.; so is 1876 piastres, $12\frac{1}{2}$ soldi, to the answer.

26. As 4s. $3\frac{1}{4}$ d. is to $5\frac{3}{4}$ lire; so is £705 " 16 " 4, to the answer.

Also,

1 Guilder or Florin = £ $\frac{1}{8}$, or 3s. 4d., Flemish.

1 Stiver - - - = 2 Pence Flemish.

1 Penning - - - = $\frac{1}{8}$ of a Penny Flemish.

The par of exchange is 36s. 7d. Flemish, for £1 sterling.

To Reduce Dutch Money into Sterling.

RULE.—As the given rate of exchange is to £1; so is the given Dutch money to the answer.

Sterling money is reduced to Dutch by the rule of Practice.

Currency is reduced to Banco by the Rule-of-Three, saying, As £100 with the agio added to it, is to 100; so is the given currency to the banco required. — And Banco is reduced to currency by reversing the terms of the analogy, saying, As 100 is to 100 with the agio added to it; so is the given banco to the currency required.

QUESTIONS FOR ILLUSTRATION.

1. Reduce 8132 guilders, 16 stivers, into sterling, exchange at 16 guilders, 8 stivers (or 34s. 8d. Flemish), per pound sterling.

G. stiv.	£.	G. st.	
As 10 " 8	1	8132 " 16	
5		5	
82		48684	£.
13		13) 10166	(782
		106	
		26	
		*	

Answer, £782 sterling.

Or thus :

	<i>d.</i>	<i>£. ster.</i>	<i>G.</i>	<i>st.</i>
As 34 " 8		1	8132	" 16
6			20	
208 stivers.			162656 stivers.	
10166				
20888				
10166				
<u>208</u>				
		= £782 sterling.		
208				
13				

2. Reduce 2279 guilders, 1 stiver, 4 pennings, currency, into banco, the agio being $3\frac{1}{8}$ per cent.

	<i>G.</i>	<i>st.</i>	<i>pen.</i>
As 103 $\frac{1}{8}$	100	2279	" 1 " 4
828	800		8
33	2	18232	" 10 " 0
			4
		3	72930 " 0 " 0
		11	24310
		<u>2210</u>	Guilders <i>banco</i> .

Or thus :

	<i>G.</i>	<i>st.</i>	<i>p.</i>
From 3	2279	" 1	" 4
11	759	" 13	" 12
Subtract	69	" 1	" 4
Answer,	2210	" 0	" 0 <i>banco</i> .
	<u>G.</u>	<u>st.</u>	<u>p.</u>

To reduce Pounds, Shillings, and Pence, Flemish, into Guilders, Stivers, and Pennings.

RULE.—Multiply the pounds and shillings by 6, adding 1 stiver more for every 2 pence Flemish, and the result will be guilders and stivers. Reduce the residue of the pence, if any, to eighths of a penny, or half farthings, for pennings.

EXAMPLES FOR ILLUSTRATION.

1. Reduce £1465 " 8 " 11 $\frac{3}{4}$, Flemish, to guilders, stivers, and pennings.

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 1465 \text{ " } 8 \text{ " } 11 \frac{3}{4} \\
 \quad \quad \quad 6 \\
 \hline
 \text{Answer, } 8792 \text{ " } 13 \text{ " } 14 \\
 \text{G.} \quad \text{st.} \quad \text{pen.} \\
 \hline \hline
 \end{array}$$

[1 $\frac{3}{4}$ d., or 1 $\frac{6}{8}$ d., = $\frac{14}{8}$, or 14 pennings.]

2. What is the amount, in Flemish money, and also in guilders, stivers, and pennings, of £188 " 17 " 6 sterling, exchange at 35s. 6d. Flemish, per pound sterling?

$$\begin{array}{r}
 \text{s.} \quad \quad \quad \text{£.} \quad \text{s.} \quad \text{d.} \\
 \left| \begin{array}{l} 20 \\ 10 \\ 5 \\ 6d. \end{array} \right| \begin{array}{l} = \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{10} \end{array} \left| \begin{array}{l} 188 \text{ " } 17 \text{ " } 6 \\ 94 \text{ " } 8 \text{ " } 9 \\ 47 \text{ " } 4 \text{ " } 4 \frac{4}{8} \\ 4 \text{ " } 14 \text{ " } 5 \frac{2}{8} \end{array} \right. \\
 \text{£. Flem.} \quad 335 \text{ " } 5 \text{ " } 0 \frac{6}{8} \\
 \quad \quad \quad 6 \\
 \hline
 \text{Guild.} \quad 2011 \text{ " } 10 \text{ " } 6 \\
 \hline \hline
 \end{array}$$

Answer, 2011 guilders, 10 stivers, 6 pennings.

QUESTIONS FOR PRACTICE.

27. How much Flemish money must be received at Amsterdam, for £3750 sterling, exchange at 35s. 5d. Flemish, per pound sterling?

28. In £500 sterling, how many guilders, exchange at 33s. 8d. Flemish, per pound sterling?

29. How many guilders will pay for a bill of exchange of £3000 sterling, at 35s. 6d. Flemish, per pound sterling?

30. In £159 sterling, how many guilders, exchange at 33s. 8d. Flemish, per pound sterling?

31. In £57 " 7 " 9 sterling, how many guilders, exchange at 35s. 6d. Flemish, per pound sterling?

32. How many guilders are equal to £871¹⁷/₈ sterling, exchange at 33s. 7d. Flemish, per pound sterling?

33. What sum must be paid at Amsterdam for a bill of £479¹⁵/₆ sterling, exchange at 34s. 9d. Flemish, per pound sterling?

34. In 2758 guilders, 14 st. 1³/₄ p., how many pounds sterling, exchange at 35s. 3¹/₂d. Flemish, per pound sterling?

35. In £886⁶/₆ Flemish currency, how many pounds sterling, the agio being at 4 per cent., and course of exchange 34s. 6d. Flemish banco, per pound sterling?

36. In £435¹⁷/₉ sterling, how much Flemish currency, the course of exchange being 33s. 10d. Flemish banco, per pound sterling, and agio 4¹/₂ per cent.?

37. Reduce 9177 guilders, 6 stivers, 14 pennings, currency of Holland, into English money, exchange at 35s. 10d. Flemish, per £1 sterling, and agio 4³/₈ per cent.

HAMBURGH.

There are here, as at Amsterdam, two sorts of money, called banco and currency; the former bears a considerable premium, the agio being from 18 to 25 per cent. — All bills of exchange are valued and paid in banco.

Occasional Directions for the Learner.

37. By the Rule-of-Five, thus:

s.	d.	£. sterling.	G.	st.	pen.
As 35	" 10 *	_____ 1 _____	9177	" 6 "	" 14
	104 ³ / ₈ *	_____ x _____	100		

Or, by Reduction, —

	£. sterling.	
As 1720*	_____	1
167*	_____	x
		1468375
		160

Accounts at Hamburgh are kept in marks and shillings, both banco and currency; also in pounds, shillings, and pence Flemish, which is an imaginary money, used only in exchange.

The par of exchange is from 34s. 7d. to 34s. 10d. Flemish, for £1 sterling, varying according to the fluctuation of the agio, and the current price of gold and silver.

THE TABLE.

3 Marks - - make 1 Rix Dollar.	[4 " 6	} Hambro' Banco and Currency.
2 Marks - - — 1 Dollar of exchange.	[3 " 0	
16 Shillings - — 1 Mark.	[1 " 6	
12 Phennings — 1 Shilling.	[0 " 1 $\frac{1}{8}$	

20 Shillings - - - make 1 Pound Flemish.	[11 " 3
12 Pence - - - — 1 Shilling Flemish.	[0 " 6 $\frac{3}{4}$
6 Phennings (banco) — 1 Groot or Penny Flem.	[0 " 0 $\frac{1}{2}$ $\frac{1}{4}$

Also,

3 Marks, Hambro' banco - - = 8 Shillings Flemish.
6 Shillings, Hambro' banco - = 1 Shilling Flemish.
1 Shilling, Hambro' banco - - = 2 Pence Flemish.

QUESTIONS FOR ILLUSTRATION.

1. Reduce £300 sterling, into marks and shillings, banco, exchange at 35s. 3d. Flemish, per £1 sterling.

s.		£.	s.	d.
		300	at 35	" 3
10	$\frac{1}{2}$	150		
5	$\frac{1}{2}$	75		
3d.	$\frac{1}{20}$	3 " 15		
		£528 " 15 Flemish.		
		20		
		10575 Shillings Flemish.		
		6		
		16) 63450 Shillings banco.		
Answer,		3965	" 10	
		<u>Marks, Shillings, banco.</u>		

2. Reduce 3965 marks, 10 shillings, banco, into sterling, exchange at 35s. 3d. Flemish, per £1 sterling.

s.	d.	£. sterling.	Marks.	s.
As 35	" 3	———— 1	———— 3965	" 10
	12			16
423			63450 shillings banco.	
			2	
		423) 126900 (300£. sterling.		
		* 00		

Answer, £300 sterling.

QUESTIONS FOR PRACTICE.

38. Reduce 8234 marks, 10 shillings, banco, into sterling, exchange at 33s. 10d. Flemish, per £1 sterling.

39. Reduce 4500 marks, currency, into sterling, exchange at 35s. 6d. Flemish, per £1 sterling, and agio 20 per cent.

40. Reduce 7276 marks, 10 shillings, 8 phennings, banco, into sterling, exchange at 34s. 5d. Flemish, per £1 sterling.

. It cannot be supposed that the extensive and complex subject of Foreign Exchanges should be fully discussed in a concise elementary Treatise on Practical Arithmetic, designed chiefly for the use of schools. But it is hoped that the various Rules already given, and the numerous Questions, both illustrative and practical, which are contained in the foregoing sketch, will enable the Learner to apply the knowledge he has acquired to the particular practice of any merchant, in whose service he may hereafter be employed. — Perhaps the most valuable work, extant in the English language, on the subject of Exchanges, is that which is published by Mr. Kelly, in one large volume quarto, entitled, ‘The Universal Cambist;’ being professedly founded on the celebrated German publication, by Kruse, called

Occasional Directions for the Learner.

38. By the Rule-of-Three, as in the solution of the second question for illustration, reducing the first term in the stating to two-pences, Flemish, and the third term to shillings, banco.

‘The Hamburgh Contorist.’ This work is universally allowed to be the most correct and comprehensive System of Exchanges that has ever yet been published. Mr. Kelly, in his ‘Universal Cambist,’ has modernised that System, and adapted it to the English standard, with many useful and important additions from unquestionable authorities.

POSITION.

Position, or the Rule of False, as it is sometimes called, is a rule by which many questions are performed that do not easily admit of a direct solution by any other rule without an algebraic process.

There are two kinds of Position, namely, Single Position and Double Position.

To Single Position belong those questions in which the results are proportional to the suppositions; such, for instance, as require the multiplication or division of the unknown number by a given number, or when it is to be increased or diminished by any proportional part of itself.

Single Position is performed by one operation only, assuming any number, (which is generally a *false* number, or one that does not answer the conditions of the question,) for the answer, and working with it, according to the terms of the question, to ascertain the result; from which result, the *true* number is afterwards found.

RULE 1.—Take any convenient number, for a supposition, and perform the same operations with it as are stated in the question to be performed with the number sought.

2. Then say, As the result of this operation is to the number supposed; so is the true result, given in the question, to the number required.

QUESTIONS FOR ILLUSTRATION.

1. A person after spending $\frac{1}{4}$ th, $\frac{1}{5}$ th, and $\frac{1}{6}$ th of his money, finds he has £161 left; what had he at first?

For the supposition, and to avoid fractions, take any number which is divisible by 4, 5, and 6, viz. 60.

Suppose the person had £60

$$\begin{array}{rcl} \frac{1}{4} & = & 15 \\ \frac{1}{5} & = & 12 \\ \frac{1}{6} & = & 10 \\ \text{Spent in all,} & & \underline{37} \\ \text{The sum left,} & & \underline{\underline{23}} \end{array}$$

Then, per rule,

$$\begin{array}{ccc} \text{£.} & & \text{£.} \\ \text{As } 23 & \text{---} & 60 & \text{---} & 161 \end{array}$$

$$\begin{array}{rcl} & 7 & \\ 60 \times \cancel{161} & = & \text{£.} \\ \hline & = & 420, \text{ the answer.} \end{array}$$

$$\begin{array}{rcl} \text{Proof.} & & \text{£.} \\ & 420 & \\ \frac{1}{4} & = & \underline{105} \\ \frac{1}{5} & = & \underline{84} \\ \frac{1}{6} & = & \underline{70} \\ & & \underline{\underline{259}} \end{array}$$

The sum left, 161, as per question.

2. Three persons purchased goods at Manchester*, amounting to £600. The second person had one-third part more than the first, and the third person, one-fourth part more than the second; what was each person's share?

* A large and populous town in Lancashire, the seat of many extensive manufactures, and, in point of population, the second town in the kingdom. It is situated on the south bank of the river Irwell, 37 miles east of Liverpool, 54 miles south of Lancaster, and 186 miles (via Macclesfield, Leek, and Ashbourn,) from London.

The adjoining town of Salford, which is situated on the north bank of the same river, is connected with Manchester by five handsome

	£.
Suppose the share of the first was	12
Then the second had	16
And the third	20
The amount,	<u>48</u>

£.	£.	£.
Then, as 48	—————	12 ————— 600

$$\frac{12 \times 600}{48} = £150, \text{ the first person's share.}$$

Hence, the second had £200, and the third person, £250.
 Now, £150 + £200 + £250 = £600, as per question.

QUESTIONS FOR PRACTICE.

1. What number is that which being increased by $\frac{1}{4}$ and $\frac{1}{5}$ of itself, will amount to 203?
2. A.'s age is double of B.'s, and B.'s is quadruple of C.'s, and the sum of all their ages is 156 years; what is the age of each?

bridges, and they now appear as one town, in the same way that London and Southwark are connected, and to which the situation of the united towns of Manchester and Salford bear a resemblance. There are twenty-six churches in Manchester and Salford, and the suburbs, belonging to the establishment; four Roman Catholic chapels; one Presbyterian chapel; five Unitarian chapels; eleven Independent chapels; two Friends' meeting-houses; twenty-nine Methodist chapels; five Baptist chapels, two New Jerusalem churches; and one Jews' Synagogue.

In the year 1821, the towns of Manchester and Salford (which constitute one entire mass of building, recognised by strangers under the general name of Manchester,) contained 22,191 houses, and 133,788 inhabitants, exclusive of Hulme, Chorlton Row, Ardwick, Chetham, and Pendleton. By the census of 1801, Manchester (including Salford) contained 84,053 inhabitants; and in the year 1831, 182,812. But the result of the census, in each of those cases, it is supposed, was considerably under the real number of inhabitants, owing to the omission of many of the back-streets, and other unexplored and obscure parts of the town.

3. A post is a $\frac{1}{4}$ in the mud, $\frac{1}{3}$ in the water, and the remainder, being 10 feet, above the water; what is its whole length?

4. A person bought a chaise, horse, and harness, for £75; the horse came to twice the price of the harness, and the chaise to twice the price of the horse and harness: what did he give for each?

5. In an orchard of fruit trees, $\frac{1}{3}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, and the remainder, which are 28, cherries; how many trees are there in all?

6. If a cardinal can pray a soul out of purgatory in an hour; a bishop in three hours; a priest in five, and a friar in seven; in what time can they pray out the same, all praying together?

7. If a lion can eat up a sheep in half an hour, a wolf in $\frac{3}{4}$ of an hour, and a dog in an hour; in what time can they devour the same, all eating together?

8. In a leaky ship, there were three pumps of different powers; the first would empty the hold of the vessel in 20 minutes, the second in 40 minutes, and the third in 60 minutes; how long would all the pumps together take in doing it?

Occasional Directions for the Learner.

8. Suppose the three pumps together would empty the hold in 10 minutes; then find what part of a unit each particular pump would discharge in that portion of time, and add the three results together. Lastly, As that amount is to 10 minutes; so is a unit, which represents the whole quantity of water contained in the hold,—to the time required. — It would be preposterous, indeed, to *suppose* the time to be 2 hours; though it is of no consequence how remote soever the supposition is from the true answer, for even that supposition, or one minute, would equally give the same answer; the operation, in the former case, would be performed by whole numbers only, without fractions.

DOUBLE POSITION.

Those questions belong to Double Position, in which the results are not proportional to the suppositions: such, for example, as require the addition or subtraction of a given number, which is no known part of the number required.

REMARK. — The rule of Position, however, will not bring out a true answer, when the number sought ascends above the first power. This is a circumstance, too, that is not always manifest from the terms in which the question is proposed. For example, suppose it were required to find the price of a horse, that was sold for £24, gaining thereby as much per cent. on the original purchase money as the horse cost. — This question could not be answered by Position. In resolving it algebraically, the result of the process is an affected quadratic equation; and it is, in fact, the same thing as to find a number, to whose *square* if there be added 100 times the said number, the sum shall be 2400. The true answer to the question is £20, being the original price of the horse, and also the gain per cent., on selling the horse for £24.

RULE 1. — Take any two convenient numbers, and perform with each the same operations as are stated in the question to be performed with the number required; then find how much the results are either more or less than the true result given in the question, and mark the difference accordingly, with the affirmative sign, +, or negative sign, —.

2. Make a large cross, like the sign of multiplication, and at the two extremities on the left side, write the numbers supposed, and opposite to them, at the other extremities of the cross, place the two errors, with their respective signs, plus or minus.

3. Multiply the numbers at each extremity, crossways, that is, the second error by the first supposition, and the second supposition by the first error.

4. If the errors are alike, viz. both plus, or both minus, take their difference for a divisor, and the difference of the products for a dividend. But if the errors are unlike, viz. one plus, and the other minus, take their sum for a divisor, and the sum of the products for a dividend.* In each case, the quotient will be the answer to the question.

QUESTIONS FOR ILLUSTRATION.

1. There is an army, to the number of which if you add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of itself, and from the amount subtract 5 000, the remainder will be 100 000; required the number of men.

First supposition.		Second supposition.	
	<i>Men.</i>		<i>Men.</i>
Supposed number,	24 000		60 000
$\frac{1}{2}$ =	12 000	$\frac{1}{2}$ =	30 000
$\frac{2}{3}$ =	16 000	$\frac{2}{3}$ =	40 000
$\frac{3}{4}$ =	18 000	$\frac{3}{4}$ =	45 000
The amount,	70 000		175 000
	5 000		5 000
The remainder,	65 000		170 000
Instead of	100 000		100 000
The first error,	<u>35 000, minus.</u>	2d error,	<u>70 000, plus.</u>

* These different operations, depending on the similarity or dissimilarity of the errors, may be easily remembered by the following lines: —

When errors are of unlike kinds,
Addition doth ensue;
But if alike, Subtraction finds
Dividing work for you.

Double Position.

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$$\begin{array}{rcl}
 24\ 000 & & 35\ 000\text{—} \\
 & \swarrow & \searrow \\
 60\ 000 & & 70\ 000\ + \\
 35\ 000 & & 24\ 000 \\
 \hline
 2100000000 & & 1680000000 \\
 105,000) & & 3780000,000\ (36\ 000, \text{ answer.} \\
 & & 630 \\
 & & * 000
 \end{array}$$

	<i>Men.</i>
Proof.	36 600
$\frac{1}{2}$	= 18 000
$\frac{2}{3}$	= 24 000
$\frac{3}{4}$	= 27 000
	<u>105 000</u>
	5 000

The remainder, 100 000, as per question.

2. What number is that, which being multiplied by 6, the product increased by 18, and the sum divided by 9, the result will be 40?

First supposition.

$$\begin{array}{r}
 9 \\
 6 \\
 \hline
 54 \\
 18 \\
 \hline
 9) \overline{72} \\
 8
 \end{array}$$

The result, 8
The first error, 32, minus.

Second supposition.

$$\begin{array}{r}
 27 \\
 6 \\
 \hline
 162 \\
 18 \\
 \hline
 9) \overline{180} \\
 20*
 \end{array}$$

Second error, 20, minus.

* Here, notwithstanding the result is 20, instead of 40, which is just one-half of what it ought to be, yet the supposition, 27, is not therefore one-half of the answer; because, in all questions belonging to Double Position, the results are not proportional to the suppositions.

$$\begin{array}{r}
 9 \quad \quad 32 - \\
 \quad \quad \quad \times \\
 27 \quad \quad 20 - \\
 32 \quad \quad 9 \\
 \hline
 54 \quad \quad 180 \\
 81 \\
 \hline
 864 \\
 180 \\
 \hline
 12 \overline{) 684} \\
 \underline{57}, \text{ the number required.}
 \end{array}$$

$$\begin{array}{r}
 \text{Proof.} \quad 57 \\
 \quad \quad 6 \\
 \hline
 \quad \quad 342 \\
 \quad \quad 18 \\
 \hline
 \quad 9 \overline{) 360}
 \end{array}$$

The result 40, as per question.

REM. All the questions which are usually answered by Position, (or Supposition, as it is denominated by some Writers,) may be solved, in a more easy and scientific manner, by Algebra.

QUESTIONS FOR PRACTICE.

1. Three persons, A., B., and C., having gained a prize of £30,000 in the lottery, it is to be divided among them in the following manner; the second person to have £900 more than the first, and the third person, £1200 more than the second; what is the amount of each person's share?

2. Three beggars, D., E., and F., upon the preliminaries of a general peace being signed, had 324 pence thrown amongst them, of which each seized as many as he could. On comparing particulars, however, it was found that E. had got 15 pence more than D., and that F. had exactly one-fifth of the money which was secured by the other two; how many pence did each severally obtain?

3. A scholar asking his master how old he was, received the following answer: "Your age is now one fourth of mine; but 8 years ago, your age was only one tenth of mine." What are the ages of each?

4. There is a fish whose tail is 9 inches, its head is as long as its tail and half its body, and its body is as long as its head and tail; required the whole length of the fish.

5. Two persons, G. and H., having each a number of sovereigns, G. says to H., "If you will give me 25 of your sovereigns, I shall have as many as you will have left;" to which H. replies, "If you will give me 22 of yours, I shall then have twice as many as you will have left:" how many sovereigns had each?

6. A person in play lost $\frac{1}{4}$ of his money, and then won 3 shillings; after which he lost $\frac{1}{3}$ of what he then had; and then won 2 shillings; lastly, he lost $\frac{1}{7}$ of what he then had, and, this done, found he had but 12 shillings remaining; what had he at first?

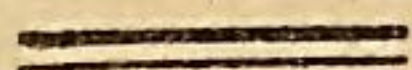
7. A man hired a labourer for 40 days, on these conditions, that for every day he worked, he should receive half a crown, but for every day he played, or was absent, he should forfeit one shilling; now at the end of the time he had to receive £2, 7, 6: how many days did he work, and how many was he idle?

8. Two persons, K. and L., play at cards; K. stakes 8 shillings, and L. 6 shillings, every game. After 28 games they leave off play, and find that they are just as they were at first; how many games did each person win?

Occasional Directions for the Learner.

5. It is manifest, from the first condition, that H. had at first 50 guineas more than G.; in the solution, therefore, suppose G.'s guineas to be any number taken at pleasure, and H.'s 50 guineas more.

9. Three companies of soldiers passing by a shepherd, the first company takes half his flock, and half a sheep; the second company takes half of what he had left, and half a sheep; and the third company takes half the remainder, and half a sheep; after which he had 20 sheep remaining: how many had he at first?



ARITHMETICAL PROGRESSION.

When a series of numbers uniformly increase by some common excess, or decrease by some common difference, they are said to be in Arithmetical Progression. Thus, 4, 7, 10, 13, 16, 19, and 22, are a series of numbers in Arithmetical Progression, the less extreme being 4, the greater extreme 22, the number of terms 7, and the common excess 3. And 63*, 56, 49*, 42, 35, 28, 21*, 14, and 7*, are a series of numbers in Arithmetical Progression, the common difference being 7, and the number of terms 9.

REM. In a series of numbers which are in Arithmetical Progression, the sum of any two extremes is always equal to the sum of any two means which are severally at equal distances from those extremes; and when the number of terms is odd, the sum of the two extremes is

Occasional Directions for the Learner.

9. Here, in order to avoid fractions, take any of the following series of numbers, for suppositions, viz. 7, 15, 23, 31, 39, 47, 55, &c.

* Certain periods in the life of man are supposed to be attended with some considerable change in the body; as the seventh year, the twenty-first, the forty-ninth, and the sixty-third; which last is called the grand climacteric. Also the eighty-first year, which is nine times nine, is commonly esteemed another grand climacteric in the life of man.

equal to twice the middle term: thus in the first series, 4, 7, 10, 13, 16, 19, and 22, the sum of 4 and 22 is equal to 7 + 19, or 10 + 16. Also, $10 + 16 = 13 \times 2$.

CASE I.

The first Term, the common Excess, and the Number of Terms being given, to find the last Term.

RULE.—Multiply the common difference by the number of terms *minus* one, and the first term added to the product will give the last term.

QUESTION FOR ILLUSTRATION.

The first term of an Arithmetical Progression is 2, the common difference 3, and the number of terms 18; required the last term

$$\begin{array}{r} 3 \\ 17 \\ \hline 51 \\ 2 \\ \hline 53, \text{ the last term.} \end{array}$$

EXAMPLES FOR PRACTICE.

1. What is the last term of an increasing Arithmetical Progression, whose first term is 1, common excess 3, and number of terms 100?

2. Suppose there are 10 pieces of cloth, the first containing 6 yards, the second 8 yards, the third 10 yards, and so on, increasing by the continual addition of 2; how many yards are there in the last piece?

3. A person bought 20 yards of cloth, to be paid for by

Occasional Directions for the Learner.

3. It is very evident, when the common difference is equal to the first term, that the common difference, being multiplied by the number of terms, will produce the last term of the series.

17 instalments, as follows, 20 pence the first week, 40 pence the second, 60 pence the third, &c.; required the amount of the last payment.

CASE II.

The first Term, the common Difference, and the last Term being given, to find the Number of Terms.

RULE. — Divide the difference of the extremes by the common difference, add one to the quotient, and the sum will be the number of terms.

QUESTION FOR ILLUSTRATION.

The two extremes of an Arithmetical Progression are 2 and 121 and the common difference is 7; required the number of terms.

$$\begin{array}{r}
 121 \\
 2 \\
 \hline
 7) \ 119 \\
 \underline{17} \\
 1 \\
 \hline
 18, \text{ the number of terms.} \\
 \hline\hline
 \end{array}$$

QUESTIONS FOR PRACTICE.

4. The first term of an Arithmetical Progression is 5, the last term 43, and the common excess 2; required the number of terms.

5. A person travelling from Edinburgh, southward, went 6 miles the first day, 9 miles the second, 12 the third, increasing every day's journey, till at last he went 66 miles in one day; how many days did he travel?

6. A nursery-man planted a nursery of firs, in the form

of an isosceles* triangle; he put one plant at the apex, three plants in the next row, five plants in the third row, and so on, in Arithmetical Progression; the last row contained 61 plants. Required the number of rows from the apex to the base of the triangle.

CASE III.

The first and last Terms, and the Number of Terms being given, to find the common Difference.

RULE. — Divide the difference of the extremes by the number of terms *minus* one, and the quotient will be the common difference.

QUESTION FOR ILLUSTRATION.

A debt was discharged at 16 different payments, in Arithmetical Progression; the first payment was £14, and the last £100. What was the common difference?

$$\begin{array}{r} 100 \\ 14 \\ \hline 5 \overline{) 86} \\ 3 \overline{) 17 \text{ " } 4} \\ \hline \underline{\underline{£5 \text{ " } 14 \text{ " } 8}}, \text{ the common difference.} \end{array}$$

QUESTIONS FOR PRACTICE.

7. A debt is to be discharged at 10 different payments, in Arithmetical Progression, the first payment to be £5, and the last £500; required the common excess.

8. A person owed a certain sum, which he paid, by regular instalments, in 13 payments, increasing in Arith-

* This word is generally pronounced, by mathematicians, according to its derivation from the Greek, with the c hard; thus, *I-sós-ke-lez*.

metical Progression; the first payment was 3 shillings, the last £5¹¹0. How much did each succeeding payment exceed the former?

9. A farmer bought 100 sheep, and gave for the first, 1 shilling, and for the last, £19¹⁷0, increasing from the first, in Arithmetical Progression, to the last. What was the common difference?

CASE IV.

The first Term, the common Difference, and the Number of Terms being given, to find the Sum of all the Terms.

RULE 1.—Find the last term of the series, by Case 1.

2. Add the two extremes together, and multiply the sum by half the number of terms, and the product will be the sum of all the terms.—Or, multiply half the sum of the extremes by the whole number of terms, and the product will be the answer.—Or, multiply the sum of the extremes by the whole number of terms, and divide the product by 2, and the quotient will be the answer.*

* The reason of this rule will appear evident, by placing a duplicate series of numbers under the given one, in an inverted order; where the sum of any two corresponding terms is equal to the sum of the first and last terms. Hence it is manifest, that the sum of the extremes multiplied by half the number of terms; or half the sum of the extremes multiplied by the whole number of terms; or half the product of the sum of the extremes by the whole number of terms, will be the true sum of the series.

Take, for example, the following series:—

8,	10,	12,	14,	16,	18,	20,	22.
22,	20,	18,	16,	14,	12,	10,	8.
—	—	—	—	—	—	—	—
<u>30</u>	<u>30</u>	<u>30</u>	<u>30</u>	<u>30</u>	<u>30</u>	<u>30</u>	<u>30</u>

$$30 \times 8$$

Consequently, $30 \times \frac{8}{2}$; or $\frac{30}{2} \times 8$; or $\frac{30 \times 8}{2}$ will severally pro-

duce 120, being the amount or sum of the given series, consisting of eight terms, viz. 8, 10, 12, 14, 16, 18, 20, and 22.

QUESTION FOR ILLUSTRATION.

The first term of a series of numbers in Arithmetical Progression is 3, the common difference 8, and the number of terms 100; required the sum of all the terms.

$$\begin{array}{r}
 8 \\
 99 \\
 \hline
 792 \\
 3 \\
 \hline
 795, \text{ the last term.} \\
 \hline
 795 \\
 3 \\
 \hline
 798 \\
 50 \\
 \hline
 39900, \text{ the sum required.} \\
 \hline
 \hline
 \end{array}$$

QUESTIONS FOR PRACTICE.

10. There are ten pieces in Arithmetical Progression, the first containing 3 yards, and the last 48 yards; how many yards were there in all the pieces?

11. There are nine pieces in Arithmetical Progression, the first containing 3 yards, and the last 43 yards; how many yards are there in all?

12. There are 11 pieces in Arithmetical Progression, the first containing 5 yards, and the last 80 yards; how many yards are there in all?

13. How many strokes does the hammer of a clock strike in the compass of 12 hours, and how many in a common year, or 365 days?

14. How many strokes do the clocks at Venice (which go on to 24 o'clock) strike in a day, and how many strokes in a common year?

15. If 100 stones be placed in a straight line, at the distance of a yard from each other, and the first a yard

from a basket; how many miles and yards will that person go, who picks them up singly, and returns with each individual stone to the basket? *

16. A sum of money was divided among twelve persons, whose shares were in Arithmetical Progression; the first received 3 shillings, the second 7 shillings, and so on: required the share of the last person, and the sum divided among them.

GEOMETRICAL PROGRESSION.

A Geometrical Progression is a series of numbers, the terms of which either uniformly increase or decrease by the constant multiplication or division of some particular number.

Thus, 4, 16, 64, 256, and 1024, are a series of numbers in Geometrical Progression, increasing by the constant multiplication of 4. And, in the same manner, 128, 64, 32, 16, 8, 4, 2, 1, $\frac{1}{2}$, and $\frac{1}{4}$, are a series of numbers in Geometrical Progression, decreasing by the constant division of 2.

REM. 1. — The common multiplier or divisor is called the ratio of the Geometrical Progression.

2. A series of numbers in Arithmetical Progression, beginning with a cipher, and increasing by the common addition of 1, being placed over the terms of any Geometrical Progression, are called the indices of those terms, by means of which, and a few of the leading terms of the Geometrical Series, any subsequent term, whose distance from the first term is given, may easily be obtained, without producing all the intermediate terms.

* This feat was once performed on the bowling-green at Lewes, in 36 minutes and a half, seemingly with great ease, by a man who lived at Dorking, in Surrey. Query, — at what rate *per hour* did he run?

CASE I.

The first Term, the Ratio, and the Number of Terms being given, to find the last Term, or any other particular intermediate Term.

RULE 1.—Write a convenient number of the leading terms of the Geometrical Progression, and over them their respective indices, placing a cipher over the first term of the Geometrical Progression, 1 over the second, 2 over the third, and so on, over the rest of the terms.

2. Observe what figures of the indices, added together, will make one less than the number of the term in the Geometrical Series, and multiply the numbers under those indices together, dividing the product by the first term of the Geometrical Series, and the result will be the term required. From this new term of the Geometrical Series, and the preceding terms, another still more remote term may be found, by observing the indices whose sum or sums will be equal to the number of the term required, *minus* one, and dividing the product of the corresponding terms in the Geometrical Series by the first term of that Series, as before. Proceed, in the same manner, till the last term of the Geometrical Series has been duly found.

REM. — It is always more convenient, for the purpose of abbreviation, to arrange the terms of the Geometrical Series which are to be multiplied together, with the sign of multiplication between them, as the numerator of a fraction, writing below, for the denominator, the first term of the Geometrical Series, when there is only one product in the numerator; the square of that term, when there are two products; the cube of it, when there are three; and so on, in proportion to the number of products, which is always equal to the number of *crosses* contained in the numerator. Then abbreviate the terms of the fraction, according to the rules already given in a former part of this work.

QUESTION FOR ILLUSTRATION.

Required the 7th, 12th, and last terms of a Geometrical Progression, whose first term is 4, the ratio 3, and number of terms 18.

Here, the seven leading terms of the Geometrical Series, with their corresponding indices, are as follows: —

0	1	2	3	4	5	6
4	12	36	108	324	972	2916.

Now, in order to find the 12th term of this Series, without producing any of the intermediate terms, we observe that the sum of the indices, 6 and 5, is just one less than the number of that term, or $12 - 1$.

Therefore, $\frac{2916 \times \overset{243}{\cancel{972}}}{\cancel{4}} = 708\ 558$, the 12th term, which was required, whose index in the Arithmetical Series, is consequently 11.

In the next place, the sum of the indices, 11 and 6, is one less than 18, the number of the last term in the Geometrical Series required.

Therefore, $\frac{708588 \times \overset{729}{\cancel{2916}}}{\cancel{4}} = 516\ 560\ 652$, the 18th term required, whose index, in the Arithmetical Series, is 17.

The same result may be obtained, by means of one operation only, if we continue the leading terms of the Geometrical Series, three places further, as follows: —

0	1	2	3	4	5	6	7	8	9
4	12	36	108	324	972	2916	8748	26 244	78 732.

Here $9 + 8 = 18 - 1$.

Therefore, $\frac{78\ 732 \times \overset{6561}{\cancel{26\ 244}}}{\cancel{4}} = 516\ 560\ 652$, the 18th term, as before.

The Operation.

$$\begin{array}{r}
 78\ 732 \times 6561 \\
 4\ 723\ 92 \\
 39\ 366\ 0 \\
 472\ 392 \\
 \hline
 \underline{\underline{516\ 560\ 652}}, \text{ the answer}
 \end{array}$$

Lastly, the same answer may be obtained, by means of only 6 leading terms of the Geometrical Series, as follows : —

0	1	2	3	4	5
4	2	36	108	324	972

First, $5 + 4 + 4 + 4 = 18 - 1$.

Therefore, $\frac{972 \times \overset{81}{\cancel{224}} \times \overset{81}{\cancel{224}} \times \overset{81}{\cancel{224}}}{\cancel{4} \times \cancel{4} \times \cancel{4}} = 972$ multiplied by the cube of 81 = $972 \times 531\ 441 = 516\ 560\ 652$, the 18th term of the series, as before.

The Operation

$$\begin{array}{r}
 531\ 441 \\
 972 \\
 \hline
 478\ 296\ 9 \\
 38\ 263\ 752\ (= 8\ \text{times}\ 9\ \text{times.}) \\
 \hline
 \underline{\underline{516\ 560\ 652, \text{ the answer.}}}
 \end{array}$$

QUESTIONS FOR PRACTICE.

1. The first term of a series of numbers in Geometrical Progression is 3, the number of terms 12, and the ratio 2; required the last term.

2. Required the 12th term of a Geometrical Series, whose first term is 3, and ratio 3.

3. A boy bought 15 oranges, and by agreement was to pay only the price of the last, reckoning a farthing for the price of the first orange, a halfpenny for the second, a penny for the third, and so on, doubling the price to the last; required the price of the oranges.

4. A person dying left 10 children, to whom he bequeathed a certain sum in the following manner; the youngest child to have £50, the second £75, the third £112.5, and so on, every child to exceed the next younger in the proportion or ratio of $1\frac{1}{2}$ to 1: what will be the fortune of the eldest?

CASE II.

The first Term, the Ratio, and the Number of Terms being given, to find the Sum of all the Terms.*

Multiply the last term by the ratio, from the product subtract the first term, and divide the remainder by the ratio *minus* one, and the quotient will be the sum of all the terms.

QUESTION FOR ILLUSTRATION.

The first term of a Geometrical Progression is 3, the ratio 4, and the number of terms 16; required the last term, and the sum of all the terms.

First,	0	1	2	3	4	5
	3	12	48	192	768	3072

Then, $5 + 5 + 5 = 16 - 1$.

Therefore, $\frac{1024 \times 1024 \times 3072}{3 \times 3 \times 3} = 3\ 221\ 225\ 472$, the 16th term.

Lastly, $3\ 221\ 225\ 472$

Subtract, 3

3) $12\ 884\ 901\ 885$

4\ 294\ 967\ 295, the sum required.

* And, consequently, the last term, when not actually given or specified in the question, may always be found by the preceding Case.

Proof, by Simple Addition.

3	1st term.
12	2nd - - -
48	3rd - - -
192	4th - - -
768	5th - - -
3 072	6th - - -
12 288	7th - - -
49 152	8th - - -
196 608	9th - - -
786 432	10th - - -
3 145 728	11th - - -
12 582 912	12th - - -
50 331 648	13th - - -
201 326 592	14th - - -
805 306 368	15th - - -
3 221 225 472	16th - - -
4 294 967 295,	the sum required.

QUESTIONS FOR PRACTICE.

5. The first term of a series in Geometrical Progression is 1, the ratio 3, and the number of terms 8; required the last term, and the sum of the series.

6. The first term of a Geometrical Series is 1024, the ratio $1\frac{1}{2}$, and the number of terms 11; required the last term, and the sum of all the terms.

7. A person sold a horse on condition of having 1 farthing for the first nail in his shoes, 2 farthings for the second, 4 farthings for the third, and so on, doubling the price of every nail to 36, the number of nails in his four shoes; what was the price of the horse?

8. A laceman agreed with a lady to sell her 20 yards

Occasional Directions for the Learner.

8. To this question Mr. Vyse has given an erroneous answer, in the Key to his Tutor's Guide, second edition *corrected*, p. 194. According to his solution, the amount of the lace would be £ 290 565¹¹7⁴; just eight times as much as it ought to be!

of lace, on condition that she gave 2 pins for the first yard, 6 pins for the second, 18 pins for the third, and so on, trebling the price of every yard; what would the lace amount to at that rate, supposing the pins to be sold at a farthing per 100?

9. A dealer in coffee, on the prospect of a contingent event, agreed for 50 guineas in advance, to return the first day, one coffee berry; the second day, two; the third, four; and so on, doubling the quantity every day, till the same took place. Supposing the time to be 60 days, what number of berries will the whole amount to, and what is their value, at 2s. 6d. a pound, supposing 1000 berries to weigh, upon an average, one pound?

10. A husbandman agreed to serve his master during harvest time, forty days, provided he would give him a barley-corn only for the first day's work; three for the second; nine for the third; and so on, in Geometrical Proportion: what would he have to receive in money for his labour, supposing there are 491,520 grains in a bushel, and each bushel to be worth 4 shillings?

11. Suppose 100 stones were placed in a straight line, the first stone being at the distance of one yard from a basket, the second stone two yards, the third four yards, the fourth eight yards, and so on, in Geometrical Progression; how many miles would the person go, who

Occasional Directions for the Learner.

10. Write the first 11 terms of the Geometrical Series progressively, and over them, their respective indices; from the two last terms (whose indices are 10 and 9) find the 20th term, whose index is 19; multiply that term by 3, and the product will be the 21st term of the Geometrical Series, whose index is 20. And from the two terms last found, whose indices are 20 and 19, may be obtained the 40th term of the Geometrical Series,—and then the sum of all the terms,—the amount in bushels,—and the value, at 4 shillings a bushel.

picks them up singly, and returns with each stone to the basket?

CASE III.

To find the Sum of any decreasing Series in Geometrical Progression, the last Term being a Cipher.

RULE. — Divide the square of the first term by the difference between the first and second terms, and the quotient will be the sum of all the terms.

QUESTION FOR ILLUSTRATION.

A large ship pursues a small one, steering the same course, at the distance of 12 miles from it, and sails twice as fast as the small ship; how many miles must the large ship sail before she overtakes the other?

Here, the first term is 12 miles, the second 6 miles, the third 3 miles, and so on, decreasing in a Geometrical Series, till the distance is 0.

Then, per rule,

$$\begin{array}{r} 12 \\ 12 \\ 6 \overline{) 144} \\ \underline{24} \text{ miles, the answer.} \end{array}$$

QUESTION FOR PRACTICE.

12. Suppose a ball to be put in motion by a force which drives it 18 miles the first hour, 15 miles the second, $12\frac{1}{2}$ miles the third, and so on, decreasing continually in the proportion of 5 to 5; what space would the ball move through?

PERMUTATION,

AND

COMBINATION.



THE Permutation of quantities is the shewing how many different ways any given number of things may be changed ; and, here, the only thing to be regarded is, the situation in which they stand, so that no two parcels may have all their quantities placed in the same order.

The Combination of quantities is the shewing how often a less number of things may be taken out of a greater number, and combined together, without considering their situation, or the order in which the particulars of each combination might be placed ; and, here, every parcel must be different from all the rest, and no two, consequently, will consist of precisely the same quantities or things.

CASE I.

To find the Number of Permutations or Changes that can be made of any given Number of Things.

RULE.— Multiply all the terms continually together, and the last product will be the number of changes required.

QUESTION FOR ILLUSTRATION.

How many changes, each consisting of 8 notes, can be rung upon 8 bells?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40\,320$ changes, the answer.

QUESTIONS FOR PRACTICE.

1. In how many different situations may 7 persons be placed, at dinner?

2. How many changes, of 12 notes each, may be rung upon 12 bells, and how many years would they take in ringing, supposing 20 changes or rounds to be rung every minute, — the year consisting of $365\frac{1}{4}$ days?

3. An accountant* told a gentleman, who had constantly eight persons at his table, that he would be glad to make a ninth, and he was willing to give him £50 for his board, so long as he could arrange the company in a different order, every day, at dinner; this proposal being acceded to, what would his board cost him *per annum*, reckoning every year to contain, on an average, $365\frac{1}{4}$ days?

4. How many different ways may the 26 letters of the English alphabet be arranged, each combination containing the whole 26 characters, but in a different order?

CASE II.

To find the Number of Compositions which may be made out of a given Number of Sets, the Number of Particulars in each Composition being equal to the Number of Sets.

RULE.—Multiply the given number of things in each set, continually together, and the last product will be the answer required.

* This word, Dr. Johnson says, was formerly written *accomptant*; but, by gradually softening the pronunciation, in time the orthography changed to *accountant*.

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QUESTION FOR ILLUSTRATION.

Suppose there are 4 companies, in each of which there are 12 men ; how many different ways may 4 men be chosen out of them, taking one man, each time, out of every company ?

$$12 \times 12 \times 12 \times 12 = 20736, \text{ the answer.}$$

QUESTIONS FOR PRACTICE.

5. Suppose there are 4 companies, in one of which there are 9 men, in another 8, and in each of the other 2 ; how many ways may 4 men be chosen out of them, each set to consist of 4 individuals belonging to the 4 different companies ?

6. How many different ways may two common dice come up, having six different faces upon each ?

7. How many different ways are there of throwing 5 dice, having 6 faces each ?

CASE III.

Any Number of different Things being given, to find how many Changes may be formed out of them, by taking a given Number of them at a Time.

RULE.—Take a series of numbers in Arithmetical Progression, beginning with the given number of things, and decreasing by 1, till the number of the terms is equal to the given number of quantities to be taken at a time ; then the continual product of those terms will be the number of changes required.

Occasional Directions for the Learner.

6. Here are, as it were, two companies, each containing 6 individuals ; and it is required to find how many ways these can be combined, taking two at a time, and one out of each company.

QUESTION FOR ILLUSTRATION.

How many changes may be rung with 4 bells out of a peal consisting of 10 bells?

$10 \times 9 \times 8 \times 7 = 5040$, the answer.

QUESTIONS FOR PRACTICE.

8. How many changes may be made out of the first five letters of the alphabet, a, b, c, d, and e, taking two at a time?

9. How many different words, consisting of 6 letters each, may be composed out of the 26 letters of the alphabet, admitting that any six letters, arranged in any order, may constitute a word?

CASE IV.

Any Number of different Things being given, to find how many Combinations may be formed out of them, by taking a certain Number of them at a Time, no Regard being had to the particular Order in which they might be arranged.

RULE 1.—Take as many terms of the Arithmetical Series, 1, 2, 3, 4, &c. as are equal to the number of things to be taken at a time, and find their continual product.

2. Take another series, consisting of the same number of terms, beginning with the number of things out of which the choice or election is to be made, and decreasing by the common difference 1, and find their continual product.

3. Divide the last product by the former, and the quotient will be the required number of combinations.

REM. — Instead of actually multiplying the terms described together, it will be more convenient first to arrange them respectively as the denominator and numerator of a vulgar fraction, and then to abbreviate the whole, as in Reduction of Vulgar Fractions.

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QUESTION FOR ILLUSTRATION.

How many combinations, of four different bells each, without regarding the order in which they could be varied, may be made out of a peal of 10 bells?

$$\frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} = 210 \text{ combinations, the answer.}$$

These combinations, however, admit of 5040 variations or changes, as appears from the Question for Illustration which is annexed to the rule for the preceding Case.

QUESTIONS FOR PRACTICE.

10. How many combinations, of two different letters in each, may be made out of the first five letters of the alphabet?

11. How many different combinations, each consisting of 6 letters, placed indiscriminately, may be formed out of the 26 letters of the alphabet?

12. A club of 12 persons agreed to meet weekly, five members at a time, as long as they could do so, without the same five persons ever meeting together again; how long would the club be able to continue their weekly associations, without violating that condition?



A CENTENARY
OF
MISCELLANEOUS QUESTIONS,
WITHOUT THE
SOLUTIONS OR ANSWERS,
SELECTED FOR
THE EXERCISE OF THE LEARNER.

1. TWENTY persons, men and women, dine together; the reckoning for each man is 8 shillings, and for each woman, 7 shillings; the amount of what they spent is £7 " 5 " 0; required the number of men and women respectively.

2. A father leaves £1600 to be divided among his 3 sons, in the following manner; *viz.*, the eldest is to have £200 more than the second, and the second, £100 more than the youngest; required the share of each.

3. A gentleman bequeaths to his 4 daughters £8600; the share of the eldest is to be double that of the second, *minus* £100; the second daughter is to receive 3 times as much as the third, *minus* £200; and the third is to receive 4 times as much as the fourth, *minus* £300; what are the respective portions of these young ladies?

4. A gentleman leaves 11 000 crowns to be distributed to his widow, 2 sons, and 3 daughters, in such proportion that the mother should receive twice the share of a

son : and each son twice as much as a daughter ; required their shares respectively.

5. A widow lady leaves 4 sons, who share her effects, as follows : the eldest takes a half, *minus* £3000 ; the second takes a third, *minus* £1000 ; the third son takes exactly a fourth of the property ; and the fourth takes a fifth of the property, *plus* £600 ; what did the widow die worth, and how much did each son receive ?

6. Required to find a number such, that if we add to it its half, the sum will exceed 60 as much as the number itself is less than 65.

7. Divide a line of 25 inches in length, into two such parts, that the longer part may be equal to 49 times the shorter part.

8. Having bought some yards of cloth after the rate of 5 yards for 7 crowns, and sold it at 11 crowns for 7 yards, I find my gain to be upon the whole 100 crowns ; how many yards did I buy in all ?

9. As I was going to St. Ives,
I met fifty old wives,
Every wife had fifty sacks,
Every sack had fifty cats,
Every cat had fifty kittens ;
Kittens, cats, sacks, and wives,
How many in all were coming from St. Ives ?*

10. One asked a shepherd to sell him 1000 sheep, who answered that he had not so many to dispose of, for he wanted of that number just as many as the number he had, half as many, and $72\frac{1}{2}$ sheep more ; how many sheep had he ?

11. The united ages of two persons being 100 years,

* As this question is usually proposed, it is a mere quirk, involving no arithmetical calculation whatever.

and the age of one of them exceeding that of the other by 40 years ; what was the age of each ?

12. A., B., and C. owe me each a sum of money, and it appears, from comparing their accounts together, that the debts of A. and B. are equal to £47 ; of A. and C. £71 ; and of B. and C. £88 ; required the debt of each person separately.

13. There are two numbers whose sum is 60, and the greater is to the less, as 3 is to 2 ; what are those numbers ?

14. There is a certain *jet d'eau* in form of a lion, the two eyes, its mouth, and sole of its right foot, being pipes conveying water to an adjoining cistern ; the right eye can fill it in 2 days, the left in 3 days, the sole of its foot in 4, while its mouth alone can fill the cistern in 6 hours ; in what time can these different pipes fill the same cistern, when they are all running together ?

15. What number is that whose $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$, exceeds itself by a unit ?

16. There are 6 sons, and the common difference of all their ages is 4 years ; now the oldest is 3 times the age of the youngest ; what are their respective ages ?

17. It is required to divide the number 84 into two parts, so that 3 times the one part may be equal to 4 times the other.

18. Divide a field, containing 60 acres, into two parts, so that $\frac{1}{7}$ of one part may be equal to $\frac{1}{8}$ of the other.

19. Two persons, speaking of the crowns they had, says A. to B., "Give me 5 of your crowns, and I shall then have as many as you ;" B. replies, "Give me 5 of yours, and then I shall have three times as many as you." How many crowns had each person ?

20. What is that number which being multiplied by 12, and 48 added to the product, as much will be produced as if the same number had been multiplied by 18 ?

21. There are two places, distant 154 miles from each other, from each of which two persons set out at precisely the same time, designing to meet upon the road. Now one of them travels at the rate of 3 miles in 2 hours, and the other, at the rate of 5 miles in 4 hours ; how far must each travel before they meet?

22. A person who travels 7 miles in 5 hours, sets out from a certain town, and 8 hours after, another person sets out from the same place, intending to overtake him, and travels at the rate of 5 miles in 3 hours ; how far did the first person travel before the second overtook him?

23. A person being asked what o'clock it was, answered, The time past noon was equal to $\frac{3}{5}$ of the time till midnight ; required the hour of the day.

24. It is required to find a number which if added to itself, and the sum multiplied by the same number, and the product decreased by the same number, and, lastly, the remainder divided by the same number, the quotient will be 13.

25. Suppose 930 trees were planted in rows ; in the first row were 17, in the next 28, and that each succeeding row contained 11 trees more than the former ; required the number of rows, and the number of trees contained in the last row.

26. A person being asked what hour of the day it was, answered, The day is, at this time, just 16 hours long, reckoning from sunrise to sunset ; now, if $\frac{1}{2}$ of the hours that are past be added to $\frac{2}{3}$ of the hours which remain, the sum will be the hour required, computing from the hour of sunrising ; what was the hour?

27. Divide the number 50 into two parts, so that $\frac{1}{7}$ of the greater part being added to 3 times the lesser part, the sum may be 50.

28. A labourer, in 40 weeks, deposited in a savings bank 28 crowns, *minus* the pay of 3 weeks, and expended, during the same period of time, 36 crowns, *plus* the pay of 11 weeks; what pay or wages did he receive *per* week?

29. If 120 men, working 12 hours a day, have, in 75 days, dug a canal, 2000 feet long, 24 feet wide, and 8 feet deep; in how many days will 180 men, by working 10 hours a-day, dig another canal, 2800 feet long, 30 feet wide, and 18 feet deep, in a soil three times more difficult to work?

30. A. seized on $\frac{2}{3}$ of a parcel of sugar-plums, but, in the scuffle, B. took $\frac{3}{8}$ of what he had seized out of his hands, and C. laid hold of $\frac{3}{10}$ more; D. then ran off with all A. had left, except $\frac{1}{7}$, which E. secured for himself.

Next, A. and C. jointly set upon B., who, in the conflict, let fall $\frac{1}{2}$ of what he had, which were equally picked up by D. and E.

B. then kicked down C.'s hat, and to work they all went anew for what it contained; A. got a $\frac{1}{4}$ of it, B. $\frac{1}{3}$, D. $\frac{2}{7}$, and C. and E. equal shares of what was left of that stock.

D. then struck $\frac{3}{4}$ of what A. and B. had last acquired out of their hands, and they with difficulty recovered $\frac{5}{8}$ of it again, in equal shares; but the other three C., D., and E., carried off each $\frac{1}{8}$ of the same.

At length, a truce being agreed upon, it was finally arranged, that the $\frac{1}{3}$ of the whole parcel, which was left by A. at first, should be equally divided amongst them; it is required to find, after this distribution, how much of the prize remained with each of the competitors, supposing the whole number of plums to be 26 880?

31. There are 480 men to be placed in an oblong, whose length and breadth together make 52; how many men must be placed in each of the two sides?

32. Cupid complained that the Muses had taken away

his arrows; Clio, said he, has taken $\frac{1}{5}$; Euterpe, $\frac{1}{12}$; Thalia, $\frac{1}{8}$; Melpomene, $\frac{1}{20}$; Erato, $\frac{1}{7}$; Terpsichore, $\frac{1}{4}$; Polyhymnia, 30; Urania, 105; and Calliope, the most spiteful of them all, has taken 360; so that now, added he, I have but 5 arrows remaining in my quiver; how many arrows had he at the first?

33. A merchant began the world with a certain sum of money, which he improved so well, that, at the end of the first year, he doubled his original stock, *minus* £100, being expended for the use of his family; he continues the same in each of the two succeeding years, doubling his stock at the commencement of the year, *minus* £100, as before. Now, at the end of the third year, he found himself just three times as rich as when he first began business; required his first stock, and the amount at the end of each of the 3 years?

34. There is an island just 73 miles in circumference, and 3 persons, X., Y., and Z., all start together, to travel the same way about it; X. travels 5 miles a-day; Y., 8; and Z., 10; when will they all come together again, for the first time, after their departure? *

* This question has not, as far as I recollect, been clearly solved in any of our common treatises on Arithmetic. Vyse's solution, in particular, which I mention as that which is perhaps most generally known, is by no means a satisfactory one. The following solution is equally plain and intelligible, and it is always pleasing to have every step in the process fully explained, so that no doubt may remain on the pupil's mind respecting the rationale of any part of the solution.

First, $8 - 5 = 3$ miles, which Y. gains of X., in one day.

And, $10 - 5 = 5$ miles, which Z. gains of X., in the same period.

Then, as 3 miles : 1 day : : 73 miles : $24\frac{1}{3}$ days, when X. and Y. will first meet. They will also meet, or come together again, at the end of $48\frac{2}{3}$ days, 73 days, $97\frac{1}{3}$ days, &c.

Also, as 5 miles : 1 : : 73 : $14\frac{2}{5}$ days, when X. and Z. will first meet.

Now, as in any multiple of these days, $24\frac{1}{3}$ and $14\frac{2}{5}$, the same parties will respectively meet again, namely, X. and Y. at the end of $48\frac{2}{3}$ days, &c. as already stated; and X. and Z. at the end of $29\frac{1}{5}$ days, at the end of $43\frac{4}{5}$ days, at the end of $58\frac{2}{5}$ days, and at the end of 73 days; it is

35. Bought $1\frac{3}{4}$ ounces of silver plate for 10s. $11\frac{1}{4}d.$; what must be given for a whole service of plate, which weighs 327 oz. 12 dwt. 9 gr.?

36. Bought 56 pieces of kerseys, each piece containing 34 ells English, at the rate of 5s. $4d.$ per ell Flemish; what did the whole come to?

37. Required the value of 85 oz. 14 dwt. 15 gr. at 6s. $6d.$ per ounce?

38. In 4712 nails of Holland, how many yards, ells English, and ells Flemish?

39. If a piece of cloth cost £10⁶8; how many yards does it contain, the ell English being worth 8s. $4d.$?

40. If $24\frac{1}{2}$ lbs. of raisins cost 9s. $2\frac{1}{4}d.$; what will 18 frails of raisins come to, each frail weighing 3 qr. $19\frac{1}{4}lb.$?

41. Bought 72 pieces of Holland for £537¹²0, at the rate of 5s. $4d.$ per ell Flemish; how many yards were there in all, and how many ells English in each piece?

42. How many yards of 5 quarters wide flannel will be sufficient to line $18\frac{7}{8}$ yards of camlet, 3 quarters wide?

43. Reduce $\frac{2}{11}$, $\frac{3}{7}$, and $\frac{17}{286}$, to their equivalent decimal fractions, to the 4th place of decimals.

44. Required the weight of a load of oak timber, (in cwt. and decimal parts of a cwt.) containing 40 cubic feet, the weight of a cubic inch of oak being $\cdot 0330946$ parts of a pound avoirdupois.

45. Two persons, A. and B., were talking of their respective ages; says A. to B., "7 years ago, I was thrice as old as you, and, 7 years hence, if you and I live, I shall

manifest that X., Y., and Z. will be all together for the first time, after their departure, at the end of 73 days. Likewise, in 73 days after that, that is, at the end of 146 days, their second general meeting will take place; and again, at the end of 219 days, 292 days, &c.

be just twice as old as you will then be:" what are their present ages?

46. A greyhound seeing a hare at the distance of 50 of his own leaps, pursues her, making 3 leaps for every 4 of the hare's, and passing over, in 2 of his leaps, as much ground as the hare does in 3 of hers; how many leaps did each make before poor puss was overtaken by the dog?

47. Required that vulgar fraction, whose numerator, being lessened by a unit, shall be equal to $\frac{2}{3}$, but the denominator, being diminished by 1, shall make the fraction equal to $\frac{5}{7}$?

This question admits of a very easy solution by an Algebraical process; and will admit of an ingenious solution by Double Position.

48. A company of men subscribed £1369 to be laid out in building, and each man contributed just as many pounds as there were individuals in the company; how much did each contribute?

49. A prize of £2000 was divided between the captain and the crew of a privateer, in the proportion of 7 to 9; what was the captain's share?

50. A dealer in tea, having 50 lb. of fine tea, worth 10s. a pound, would mix a coarser sort with it, at 7s. a pound; what quantity of the latter sort must he use, so as to afford the whole together at 9s. per pound?

51. A farmer would mix wheat at 4s. a bushel, with rye at 2s. 6d. a bushel, so that the whole mixture may consist of 90 bushels, and be afforded at 3s. 2d. a bushel; how many bushels of each sort must be taken?

52. A person bought 30 lb. of sugar, of two different sorts, and paid for the whole, 19 shillings; the better sort cost 10d. per pound, and the inferior 7d.; how many pounds were there of each sort?

53. Two persons, M. and N. raise a joint stock of £1500, by which they gain £480, and M. had for his

share of the profits £96 more than N.; what did each person advance at the first, and what were their respective shares of the gain?

54. What is the sum of 1, 3, 5, 7, 9, 11, &c. continued to the 60th term?

55. A gentleman left his whole estate or property amongst his four sons; the eldest son had $\frac{1}{2}$, *minus* £800, the second, $\frac{1}{4}$, *plus* £120, the third had half as much as the eldest, and the youngest, $\frac{2}{3}$ of what the second had; what was the amount of the whole property, and how much was the share of each son?

56. Required the sum of 1, 3, 9, 27, 81, &c. continued to the 40th term in the series?

57. Two workmen, A. and B., were employed together for 50 days, at 5s. per day each; during that time, A., by spending only 6d. a day less than B., had saved twice as much as B., besides the expence of two days over and above; what did each person spend per day?

58. A draper having a piece of cloth which cost him 3s. 2d. a yard, sold $\frac{1}{3}$ part of it at 4s. per yard, $\frac{1}{4}$ at 3s. 8d. per yard, $\frac{1}{5}$ at 3s. 6d. per yard, and the remnant at 3s. 4d. per yard; his gain upon the whole was 15s. 2d.: how many yards did the whole piece contain?

59. A farmer having 28 bushels of barley at 2s. 4d. per bushel, would mix it with rye at 3s. per bushel, and wheat at 4s. per bushel, so that the whole quantity may consist of 100 bushels, and be worth 3s. 4d. per bushel; how many bushels of wheat and of rye must he mix with the barley?

60. A. and B., working together at the same labour, can earn 40s. in 6 days: A. and C. together can earn 54s. in 9 days; and B. and C. 80s. in 15 days; find what each person alone can earn in one day.

61. If A. and B. together can perform a piece of work in 8 days, A. and C. in 9 days, and B. and C. in

10 days; how many days will it take each person alone to perform the same piece of work?

62. If A., B., and C. can together finish a piece of work in 9 days; A., B., and D. together, in 10 days; A., C., and D. together, in 11 days; and B., C., and D., in 12 days; in what time can they all four, working together, complete the same?

63. A bookseller sold me 10 books, at a certain price, and some time afterwards, 15 books more, at exactly the same price per volume; but for the latter, I paid him 35s. more than for the former parcel: what did I pay for each parcel?

64. A draper bought 3 pieces of cloth, which together measured 159 yards. The second piece was 15 yards longer than the first; and the third, 24 yards longer than the second: what was the length of each piece?

65. Two soldiers in the last war, having obtained a certain booty, find that they have got between them 35 Napoleons; and that if one of them had not lost 4 Napoleons by the way, he should have had twice as many as the other; how many had each?

66. How many revolutions round their common centre do the hour hand, minute finger, and second finger of a clock make respectively in the course of 24 hours? and how many revolutions do they severally make in 4 years?

67. A mercer having cut 19 yards from each of 3 equal pieces of silk, and 17 from another of the same length, found that the remnants taken together were 142 yards; what was the length of each piece?

68. Bought 12 yards of cloth for £10^{..}14^{..}0. I gave 19s. a yard for part of it, and 17s. a yard for the remainder; how many yards of each were bought?

69. A courier, who travels 60 miles a day, had been dispatched 5 days, when a second messenger was sent to overtake him; in order to accomplish which, *he* tra-

velled 75 miles a day ; in what time will he overtake the first courier ?

70. A. and B. began trade with equal stocks or capitals. In the first year, A. trebled his capital, and had, besides, £27 to spare ; B. in the same time doubled his capital, and had £153 to spare. Now the amount of both their gains was just 5 times the stock with which each began business ; required the amount of that stock ?

71. The corn produced by a certain field measured only 48 quarters, being only $\frac{1}{3}$ part more than the seed sown ; how much was that ?

72. A. and B. severally cut or divided packs of cards, each consisting of 52 in number, and each took off more than he left. Now it happened that A. took off twice as many as B. left, and B. took off or lifted from his pack seven times as many as A. left ; how were the packs cut by each person respectively ?

73. A gentleman gave £98 to 3 persons. The second received $\frac{5}{8}$ of the sum given to the first ; and the third $\frac{1}{5}$ of what the second had ; what did each person receive ?

74. Being sent to market to buy a certain quantity of meat, I found that if I bought beef, at 6*d.* a pound, I should lay out all the money I had ; but if I bought mutton, which was then only 5 $\frac{1}{2}$ *d.* a pound, I should have 2 shillings left ; what was the quantity of meat sent for ?

75. A. and B. made a joint stock of £833, which after a certain time produced a clear gain of £153. Of this sum, B. had £45 more than A. ; how much did each person contribute to the original stock ?

76. A gentleman bestowed in charity £46 ; one portion of it was given, in equal parts, to 5 poor men, and the residue, also, in equal parts to 7 poor women. Now a man and a woman received between them £8 ; what

portion of the original donation was given to the men, and how much was the amount of the women's share?

77. Suppose that for every 10 sheep a farmer kept, he should be allowed to plough an acre of land, and have one acre of pasture for every sheep; how many sheep may the farmer keep who farms 700 acres?

78. A bookseller sells me two books, one containing 100 sheets, for 10 shillings; the other 50 sheets, for 6 shillings; the same price being charged for each for the binding; what was that price?

79. A gentleman wished to inclose a piece of ground with palisadoes, and found that if he set them a foot asunder, he should have too few by 150; but if he set them a yard asunder, he then should have too many by 170; how many palisadoes had he?

80. A servant agreed with a farmer to serve him 10 years, on condition that he should receive for his wages, at the end of the first year, one pint of corn; 8 pints, or one gallon, at the end of the second year; one bushel, at the end of the third year; and so on, increasing in an eight-fold proportion, to the end of the last or tenth year; what would his wages for the ten years amount to, supposing the corn to be sold for 10 shillings per bushel?

81. It is required to arrange the 9 digits, in the form of a Magic square, so that the sum of each row, taken vertically, horizontally, and diagonally, downwards or upwards, shall be always the same number.*

* The following note on Magic Squares is here subjoined for the information of those who may be inclined to investigate the subject, and to see the various processes for their construction accurately described, and clearly demonstrated. The most elaborate treatise I have ever met with, on this curious and interesting subject, may be found in the *Arithmetical Preceptor*, which was published at Sheffield, in the year

82. If 10 apples cost a penny, and 25 pears cost as much as 20 apples; and I buy 100 of both apples and pears for $9\frac{1}{2}d.$: how many were there of each sort?

83. A countryman being employed by a poulterer to drive a flock of geese and turkeys to town, in order to distinguish his own, he pulled 3 feathers out of the tail of each turkey, and one feather only out of the tail of each goose, and on counting them separately, he found the number of turkey's feathers exceeding twice the number of the geese, and 15 over and above. After this, he sold 15 turkeys and bought 10 geese; and when he drove them into the poulterer's yard, he found that the number of geese exceeded the number of turkeys, in the

1813, by the late Mr. Joseph Youle, Master of the Boys' Charity School in Sheffield. If we take any series of numbers, in arithmetical progression, commencing with the digit *one*, and terminating in any square number, the terms of which such a progressive series consists, may be so disposed in the different cells of a geometrical square, that the amount of each row, taking horizontally from side to side, or vertically from top to bottom, or diagonally from angle to angle, shall be precisely the same. Such Squares have been called Magic Squares.

The construction of these squares may afford some amusement to young persons in a winter's evening, and conduce, perhaps, to their attainments in the prosecution of other more useful and practical pursuits. Magic squares, Mr. Youle observes, were held in great veneration by some of the ancient philosophers, and were regarded as magical, on account of the supposed singular virtues which were ascribed to them, in the times of ignorance and superstition, by pretended fortune-tellers and conjurors. But modern mathematicians have long rejected this absurd notion of the efficacy of these squares, and their pretended fascinating powers are now, for the most part, entirely exploded. The construction of these squares, however, forms a most ingenious subject for speculation, and is by no means unworthy the attention of the curious. See, also, Hutton's Mathematical Dictionary, *the last edition*, which contains a very simple method of constructing a great variety of these squares, first communicated to the Author of that most valuable work, by the late Mr. Joseph Bibby Wise, of Maidenhead, Berkshire.

proportion of 7 to 3 ; required the number of each at his first setting out ?

84. A cistern into which water was conveyed by two pipes of different calibre, A. and B., will be filled in 12 hours by both of them running together, and by the pipe A., running alone in 20 hours ; in what time will it be filled by the pipe B. alone ?

85. What is the value of a wedge of gold, weighing 15 lb. 10 oz. 18 dwt. 18 grs., at £5.15.6, per ounce ?

86. A. sold B. 60 dozen of penknives, at 7s. 9d. per dozen, and 45 dozen at 10s. 6d. per dozen ; for which he received 12 yards of broad cloth, and £32 in money ; what was the cloth valued at per yard ?

87. Required the square root, to 5 places of decimals, of $\frac{1}{8}$ of $371\frac{3}{4}$?

88. A shepherd being asked how many sheep he had, replied that $\frac{2}{3}$ of $\frac{1}{5}$ part of them were 18 ; how many sheep had he ?

89. How many yards of broad cloth, at £1.5.9 per yard, besides £52.10.0 in money, must be given in barter for one ton of sugar, at $10\frac{1}{2}d.$ per pound ?

90. At how many minutes past 3 o'clock will the minute finger come up with the hour index ? Also when will these indices form a right angle with each other ; and when will they be in direct opposition, so as to form one and the same straight line ?

91. A gentleman walking out looked at his watch, and found the hour and minute fingers exactly together, between the hours of 5 and 6. At his return, having walked at the rate of 3 miles an hour, the indices were again together, between 7 and 8 o'clock ; how many miles had he walked ?

92. A pedestrian leaves Manchester at 6 o'clock in the morning, and walks $3\frac{1}{2}$ miles an hour ; at half-past 9, a messenger follows him, who travels at the rate of $4\frac{3}{4}$ miles

per hour: how many miles will he have gone when he overtakes the first?

93. A stationer sold Dutch pencils at 2s. 6d. per hundred, by which he gained 25 per cent.; but afterwards, in consequence of the war, becoming very scarce, he raised the price to 8s. per hundred: what did he gain per cent. by the advanced price?

94. A piece of ground, containing just a quarter of an acre, is to be divided into 3 gardens; the second is to contain $\frac{5}{8}$ of the first, and the third, $\frac{3}{5}$ of the second: how many perches will each contain?

95. Bought a quantity of cloth for as many pence per yard as there were yards in the whole quantity; and, by selling it again, at 16 per cent. profit, I received for the whole, £302.1.8: how many yards were there?

96. At a meeting of gentlemen, farmers, tradesmen, cottagers, and labourers, each gentleman paid 20 shillings; each farmer, 8 shillings; each tradesman, 6 shillings; each cottager, half-a-crown; and each labourer, one shilling. There were three times as many farmers as gentlemen; two more tradesmen than farmers; five more cottagers than tradesmen; and fifteen more labourers than cottagers. Now, had the company been all tradesmen, the reckoning would have amounted to £8.7.6 more; required the sum spent, and the number of persons of each class?

97. The sum of £351.5.0 was divided amongst a certain number of men, the common difference of whose shares was £1.7.6. The greatest share was £30.12.6; required the least share, and the number of men?

98. The planet Mercury, which is the nearest to the sun, completes one revolution round that luminary, about 230 000 000 miles, in 87 of our days, and 23 hours; at what rate does that primary move per second?

99. If any sum of money be put out at compound inte-

rest, at 5 per cent.; in what time will the principal be doubled?

100. Required the least number that can be divided by 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, and 30. without a remainder.

* * * *The Answer to the 62nd Question is not correct, in the Key, the true Answer being $7\frac{599}{763}$.*

In like manner, the correct Answer to the 85th Question is, £1102 " 13 " $3\frac{1}{4}$

THE END.

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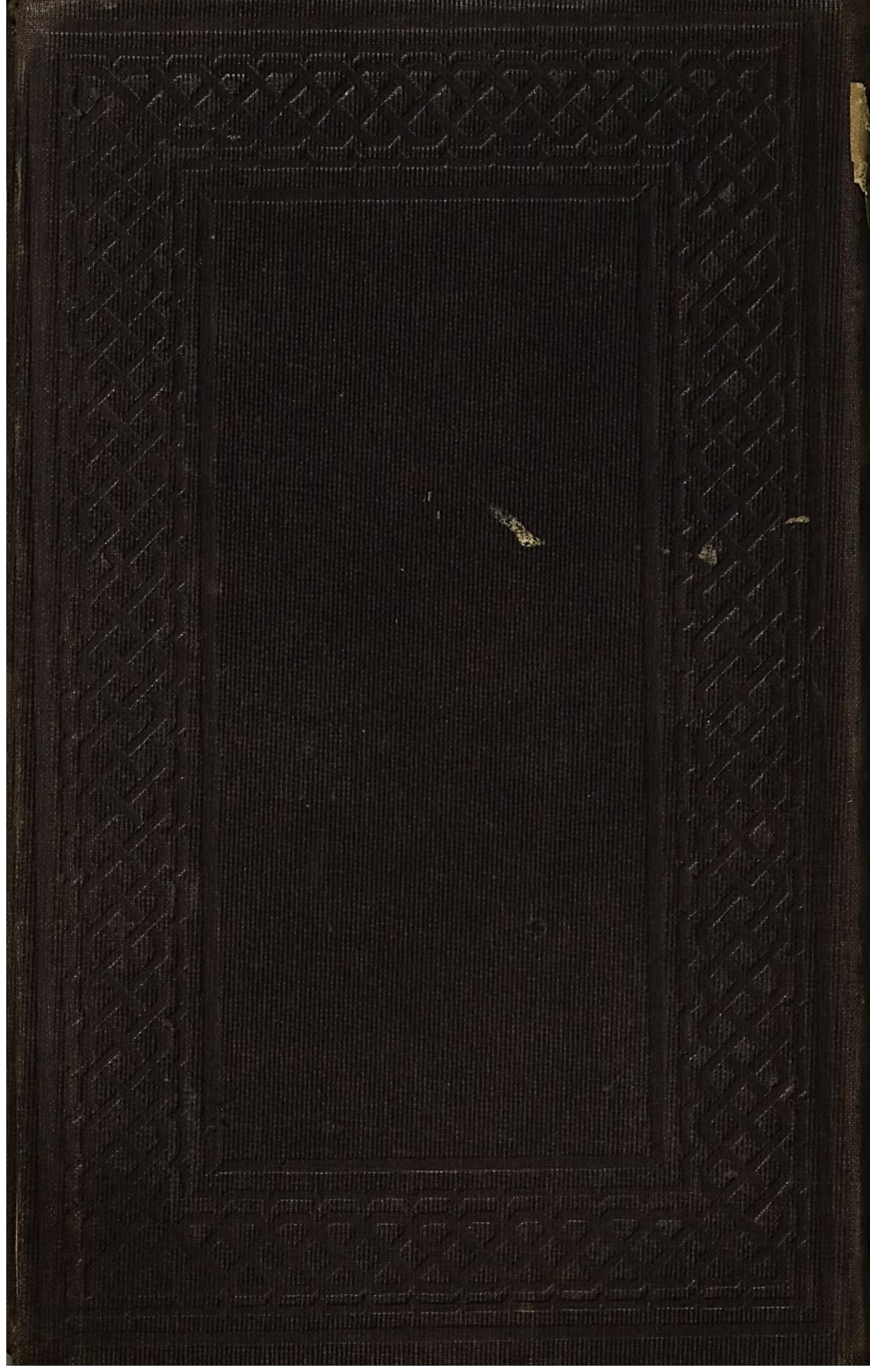
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